

FOCAL PROPERTIES OF SURFACES OF THE
SECOND ORDER.

Die Focaleigenschaften der Flächen zweiter Ordnung. Ein neues Kapitel zu den Lehrbüchern der analytischen Geometrie des Raumes. Von Professor Dr. OTTO STAUDE. Leipzig, Teubner, 1896. 8vo, viii + 186 pp.

THIS is a text-book which has grown from four articles by the author, published in : *Berichte der K. Sächsischen Gesellschaft der Wissenschaften zu Leipzig*, 1882, p. 5, Mar. 3 ; 1895, p. 483, July 1 ; *Mathematische Annalen*, vols. 20 and 27.

The focal properties of the conic in the plane are so familiar that it is natural to enquire if there be not their counterpart in space. Investigations in this line for certain particular theorems were published by : Dupin (1813), *Développement de géométrie* ; Steiner (1826), *Crelle* 1 ; Jacobi (1834), *Crelle* 12 ; Chasles (1835), *Mémoires de l'Institut*, and (1837), *Notes* ; MacCullagh (1836), *Proceedings of the Irish Academy* ; Lamé (1837), *Liouville* 2 ; Salmon (1842) ; Amiot (1843), *Liouville* 7, and (1845), 10 ; Plücker (1846), *System der analytischen Geometrie des Raumes* ; Liouville (1847), *Liouville* 12 ; Townsend (1848), *Cambridge and Dublin Mathematical Journal* 3. This list of names makes it all the more strange that these masters of geometric analysis did not carry through the reasoning and find the focal properties. It was reserved for the author to simplify much that had gone before and to develop in a simple way the general theory by the use of the now common idea of a movable coördinate trihedron. This development is also particularly valuable in that it uses throughout either the elliptic or parabolic coördinates, giving a most excellent presentation of these systems. And not the least important feature of the book is the care with which these exceptional cases are treated as part of the organic whole.

The first step in the development consists in using the equation to the cone from any point in space to each of the focal conics of a confocal system. This is found to be as follows : λ, μ, ν being the parameters of the three surfaces of the system through the point P ; ξ, η, ζ , coordinates of any point referred to the three normals at P to these surfaces as axes ; β, γ , the parameters corresponding to the two focal conics ; then the equations of the cones from P (λ, μ, ν) to the two focal conics are derived in the form (either for a central or a parabolic system) :