and let $\dfrac{\partial^2 f(x,y)}{\partial y \partial x}$ denote $\dfrac{\partial}{\partial y}\left(\dfrac{\partial f(x,y)}{\partial x}\right)$.

Let (x_0, y_0) be any point for which the conditions of the theorem are fulfilled and let the lines $x = a$, $x = b$, $y = c$, $y = d$ bound a region of the plane enclosing the point (x_0, y_0) and so small that the conditions stated are satisfied throughout the interior of the rectangle and on its boundary. Under these conditions we have

$$\int_c^d dy \int_a^b dx \,\frac{\partial^2 f(x,y)}{\partial x \partial y} = f(b,d) - f(b,c) - f(a,d) + f(a,c),$$

$$\int_a^b dx \int_c^d dy \,\frac{\partial^2 f(x,y)}{\partial y \partial x} = f(b,d) - f(a,d) - f(b,c) + f(a,c).$$

But, under the conditions of the theorem,

$$\int_a^b dx \int_c^d dy \,\frac{\partial^2 f(x,y)}{\partial y \partial x} = \int_c^d dy \int_a^b dx \frac{\partial^2 f(x,y)}{\partial y \partial x}.$$

Hence $$\int_c^d dy \int_a^b dx \left(\frac{\partial^2 f(x,y)}{\partial x \partial y} - \frac{\partial^2 f(x,y)}{\partial y \partial x}\right) = 0.$$

Now, if a function, continuous in the neighborhood of a point (x_0, y_0), is such that its integral, extended over *any* rectangle enclosing this point , is zero, it is readily seen that the function cannot be positive or negative at the point (x_0, y_0). Hence

$$\frac{\partial^2 f(x,y)}{\partial x \partial y} - \frac{\partial^2 f(x,y)}{\partial y \partial x} = 0$$

at the point (x_0, y_0). But this was any point, and the theorem is proved.

SOME OBSERVATIONS ON THE MODERN THEORY OF POINT GROUPS.

BY MISS FRANCES HARDCASTLE.

THE origins of the theory of point groups are to be found in Brill and Noether's classic memoir (see infra) published nearly twenty-five years ago, but it is only within the last fifteen years that systematic attention has been given to the subject by the Italian mathematicians, Segre, Bertini,