

3. A mark for each face, and a list of the edges and vertices in their order upon the boundary of each face.

Such a notation must contain a mark of distinction for the two sides of an edge; an easy matter if the direction of positive rotation be adopted uniformly in listing arrangements about the vertices and faces respectively.

These processes, and the proved existence of fundamental polygons, open a range of particular problems of considerable interest. But of even superior interest must be, at least until it is solved, *the problem of finding a method for constructing, a priori, upon a given surface the exceptional (Davis) special reticulations whose characteristics are given by the restrictive tables.*

NORTHWESTERN UNIVERSITY,  
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## SYSTEMS OF SIMPLE GROUPS DERIVED FROM THE ORTHOGONAL GROUP.

BY DR. L. E. DICKSON.

1. IN the February number of the BULLETIN I determined the order  $\omega$  of the group  $G$  of orthogonal substitutions of determinant unity on  $m$  indices in the  $GF[p^n]$  and proved that, for\*  $p^n > 5$ ,  $p \neq 2$ , the group is generated by the substitutions

$$O_{ij}^{\alpha\beta} : \begin{cases} \xi'_i = \alpha\xi_i + \beta\xi_j, \\ \xi'_j = -\beta\xi_i + \alpha\xi_j, \end{cases} \quad (\alpha^2 + \beta^2 = 1).$$

The structure of  $G$  was determined for the case  $p = 2$ . I have since proved† that for every  $m > 4$  and every  $p^n > 5$  of the form  $8l + 3$  or  $8l + 5$ , the factors of composition of  $G$

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\* The fact that  $p^n = 3$  is an exception was not pointed out in the BULLETIN. In fact Jordan had not proven case 2° of § 211 when  $-1 =$  square, so that the case  $a^2 = b^2 = c^2 = \dots = 1$  was unsolved when  $p = 3$ ,  $m = 3k + 1$ . The theorem is readily proven when  $p^n = 3^n$ ,  $n > 1$ ; but for  $p^n = 3$  an additional generator is necessary and sufficient, viz.,

$$W = \begin{pmatrix} 1 & 2 & 2 & 2 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{pmatrix}, \quad W^3 = 1.$$

† A preliminary account was presented before the Mathematical Conference at Chicago, December 30, 1897.