THE THEOREMS OF OSCILLATION OF STURM AND KLEIN. (SECOND PAPER.)

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THE following pages form a continuation of §§ 1, 2 of a paper presented under the same title to the Society at its December meeting and printed on pp. 295–313 of the present volume of the BULLETIN. This paper will be referred to for brevity as *Th. of Osc. 1.* No use is here made of the results contained in § 3 of the paper just mentioned, these results being merely very special cases of some of those now given.

In § 1 of the present paper, after proving two simple theorems of Sturm, by a method different from that used by him, I have used them to throw Sturm's theorem of oscillation into a slightly generalized form. In § 2 I have proved Klein's theorem of oscillation in a very general form, although as I have restricted myself here, as in the previous paper, to the case in which the coefficients of the binomial differential equations are continuous the cases here considered are not yet the most general ones. I expect in a subsequent paper to come back to these more general cases which do not seem to present any serious difficulty.

The truth of the theorems proved in § 2 (at least in important special cases) has been suspected by Klein partly from analogy with the special cases in which the differential equation has a polynomial solution, and partly from analogy with the simple cases of Lamé's equations (I use the term here in Klein's most general sense) with two or at most three variable parameters, in which cases he had given rough geometrical proofs which however made no pretense at rigor.* In the more complicated cases no proof of any sort has as far as I know been given.

§ 1. *Sturm's Generalized Theorem of Oscillation.*

We will assume that in the interval $a \leqq x \leqq b$ $p(x)$ and $q(x)$ are single valued continuous real functions of the real

*Cf. Klein's lithographed lectures: "Ueber lineare Differentialgleichungen der zweiten Ordnung," p. 401–431. We shall see that the restriction which Klein here makes to equations which are everywhere regular is unessential.