established for these cases. On the other hand the theorem of oscillation for other differential equations which like Lamé's involve two parameters* may be established by reasoning almost identical with that here used, the difference again coming in only in the four Lemmas. I hope soon to return to these and other similar questions.

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SOME EXAMPLES OF DIFFERENTIAL INVARIANTS.

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IN the following paper certain invariants for projective transformations are given. The derivation, according to Lie's methods, is given in full for the plane, and the method for the corresponding problem in space of three dimensions is sketched in, and the results of the solution are given. It is believed that all the invariants given are new.

For an infinitesimal point transformation of the xy plane x and y receive the increments

$$\delta x = \xi (x, y) \delta t, \qquad \delta y = \eta (x, y) \delta t,$$

respectively, where δt is an infinitesimal independent of x and y. This infinitesimal transformation is represented by the symbol

$$Xf = \xi (x, y) \frac{\partial f}{\partial x} + \eta (x, y) \frac{\partial f}{\partial y}.$$

The increment of any function $\varphi (x, y)$ is then

$$\delta\varphi = \frac{\partial \varphi}{\partial x} \delta x + \frac{\partial \varphi}{\partial y} \delta y = \left(\frac{\partial \varphi}{\partial x} \xi + \frac{\partial \varphi}{\partial y} \eta\right) \delta t = X\varphi \cdot \delta t.$$

If, then, φ is to be invariant for the transformation Xf, we have as a necessary and sufficient condition $X\varphi = 0$. Lie

*For instance Lamé's generalized equation. See Reihenentwickelungen, p. 125.