

complementary with respect to n of the given combination. Suppose now we form the n_m combinations of any n numbers m at a time and consider the set of combinations formed by combining each of these n_m combinations with their complementaries in such a way that the numbers in any combination arranged in their natural order are immediately followed by the numbers in the complementary combination arranged in the same way. The paper then gives an expression for the number of inversions in any combination of this set and also the excess of the number of combinations in the set which have an even number of inversions over those which have an odd number.

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THE THEOREMS OF OSCILLATION OF STURM AND KLEIN. (FIRST PAPER.)

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IN the first volume of *Liouville's Journal* (1836) Sturm has deduced certain properties of the real solutions of linear differential equations of the second order which are of fundamental importance both in pure and in applied mathematics. The opinion has been expressed* that Sturm's work cannot be regarded as rigorous and that other methods must be substituted for his, for instance the method of successive approximations recently employed by Picard for establishing some of these theorems. In one sense it is true that Sturm's work is not rigorous, as hardly any work in analysis done during the first half of the present century shows an appreciation of the difficulties connected with the conception of continuity. The work of Sturm may, however, be made perfectly rigorous without serious trouble and with no real modification of method. In the first two sections of the present paper I have proved such of Sturm's results as are necessary to establish his theorem of oscillation.† In doing this I have departed somewhat from his

* Cf. the first paragraph of Picard's note in the *Comptes Rendus* for February, 1894, and also Klein, *Lineare Differentialgleichungen der zweiten Ordnung* (lithographed 1894) p. 266: "In der That genügen die Existenzbeweise, wie sie Sturm und Liouville führen, keineswegs den heutigen Anforderungen der Strenge. Man wird verlangen alle die von ihnen gegebenen Entwicklungen in neuer Weise abzuleiten."

† This name is due to Klein.