In a similar way we see that $\varphi\ (x)$ can have no complex root whose pure imaginary part is negative.

$x_1, x_2, \cdots, x_k$ are, therefore, all real. Suppose one of them were greater than e_n. Call this one (or, if there are more than one, the greatest of them) x_1. Then the above equation again involves a contradiction since no term is negative or zero.

In the same way we see that no root can be less than e_1.

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INFLEXIONAL LINES, TRIPLETS, AND TRIANGLES ASSOCIATED WITH THE PLANE CUBIC CURVE.

BY PROFESSOR HENRY S. WHITE.

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THE configuration of the nine inflexions of a nonsingular plane cubic and the twelve lines containing them three-and-three would seem too well known to merit discussion. It is the uniform mode, in such compends as I have seen, to show first that every line joining two inflexions meets the cubic again in a third inflexion; second, that through the nine inflexions there must lie in all twelve such lines; and thirdly, that three lines can be selected which contain all nine inflexions. These three lines are termed an inflexional triangle, and the entire twelve are thought of as constituting four inflexional triangles. But there is another arrangement of the nine lines remaining after the erasure of one inflexional triangle, which I have not happened to find mentioned, which yet seems the easiest and most natural mode of access to the inflexional triangle itself.

It shall be presupposed known that there are nine inflexional points, and that every line joining two of them contains also a third. Select two inflexional points A, B, and any third C not collinear with the first two. Call these three an *inflexional triplet.* Join them by three lines, and produce BC, CA, AB to meet the cubic in a second inflexional triplet, in the points A_1, B_1, C_1 respectively.*

Repeating the process upon these three, determine a third triplet A_2, B_2, C_2. From these, determine similarly a fourth triplet. Since its points cannot be additional inflexions, nine having been included already; and since they cannot be the points of the second triplet (as is evident from the figure) unless certain inflexions coincide, they must be the

*It is easily seen that A_1, B_1, C_1, and again A_2, B_2, C_2, are not collinear.