THE ROOTS OF POLYNOMIALS WHICH SATISFY CERTAIN LINEAR DIFFERENTIAL EQUATIONS OF THE SECOND ORDER.

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IN volume 6 of the *Acta Mathematica*, Stieltjes has given a remarkable method for showing in how many different ways certain parameters in an important class of linear differential equations of the second order can be so determined that the equation shall have a polynomial solution, and in the course of the work the position of the roots of these polynomials is determined. It sometimes happens that the answer to the first part of the problem here referred to can be obtained more easily or more naturally by other methods. For instance in the case of the hypergeometric differential equation the forms in which solutions can be expanded in series show us at once in what cases we have a polynomial solution; and in the case of Lamé's equation the theorem of oscillation leads us most naturally (from some points of view) to the result.* There still remains the second part of the above problem, viz.: the determination of the position of the roots of the polynomials. The method of Stieltjes is connected with a problem in the equilibrium of particles on a straight line. By generalizing these considerations so as to bring in particles lying in a plane, we can, as I have shown,† obtain a theorem concerning the position of the roots of the polynomials, which, though in itself less far reaching (in some respects) than that of Stieltjes, gives us in the cases above referred to the information we want. I should like here to emphasize three points:

1. This method enables us to avoid the determination of an upper limit to the number of determinations of the parameters which give polynomial solutions.

2. It enables us to go beyond the cases considered by Stieltjes inasmuch as the singular points of the differential equation may now be complex.

3. Owing to the relatively small result we wish to attain

*Cf. my book: Ueber die Reihenentwickelungen der Potentialtheorie, Leipzig, Teubner, 1894. See pages 210–213.

†See pages 215–216 of the book just referred to.