CERTAIN CLASSES OF POINT TRANSFORMATIONS IN THE PLANE.

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A POINT transformation is an operation by which a point is carried into the position of some point. As far as the general definition is concerned the path described by the point and the time consumed in the change of position are immaterial, accordingly the coördinates of the final position of the point are functions only of the coördinates of its initial position, and a point transformation of the xy-plane into itself is represented analytically by two equations of the form.

$$x_1 = X(x, y), \quad y_1 = Y(x, y), \tag{1}$$

where the functions X and Y are independent analytic functions in the Weierstrassian sense.

By such a transformation point is transformed into point, lineal element* into lineal element, curve into curve, intersecting curves into intersecting curves, curves in contact into curves in contact. By imposing geometrical conditions on the transformation, there result corresponding analytical conditions for the determination of the forms of the functions X and Y and thus particular categories of point transformations arise.

For example, if the transformation (1) is to change straight line into straight line, or in other words, to leave the ordinary differential equation of the second order

$$y'' = 0$$

invariant, the functions X and Y are found to have the forms

$$X \equiv \frac{a_1x + b_1y + c_1}{a_3x + b_3y + c_3}, \quad Y \equiv \frac{a_2x + b_2y + c_2}{a_3x + b_3y + c_3}, \tag{2}$$

which define the *general projective transformation* of the xy-plane. If, further, the point transformation is to transform parabola into parabola, or what amounts to the same

* The term lineal element is here used in the sense introduced by Lie, namely, to designate the ensemble of a point and a straight line through the point.