CONDITION THAT THE LINE COMMON TO $N-1$ PLANES IN AN N SPACE MAY PIERCE A GIVEN QUADRIC SURFACE IN THE SAME SPACE.

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THIS note is a generalization of a proof given in a recent paper * of the geometric significance of the sign of a certain determinant. This determinant was the combinant of four linear spherical complexes; the spheres common to the four complexes are real when the combinant is negative.

When applied to linear line complexes, which can be derived from the spherical by an imaginary transformation, I subsequently found, by another method,† that the corresponding determinant is positive when the lines which cut four given ones are real.

The law is general, and will apply to determinants of odd order, and to imaginary transformations.

(1) Let $$\sum_{i=1}^{n+1} a_{i,k}\, x_i = 0 \qquad [k = 1, 2, \cdots, n \cdots 1]$$

represent $n-1$ linear equations, homogeneous in $n+1$ variables x_i; these can be regarded as the equations of $n-1$ planes in space of n dimensions.

Let the variables x_i satisfy the homogeneous quadratic equation

$$\varphi\,(x_1, x_2, x_3, x_4, x_5, x_6) = \varphi\,(x) = 0 \tag{2}$$

which may be regarded as the equation of a quadric surface in the same space. The $n-1$ planes will intersect in a line; I propose to give the criterion for the reality of the two points in which this line pierces the given surface. It depends upon the sign of a determinant which may be defined as follows:

Let

$$y_1 \frac{\partial \varphi}{\partial x_1} + y_2 \frac{\partial \varphi}{\partial x_2} + \cdots + y_{n+1} \frac{\partial \varphi}{\partial x_{n+1}} = 0 \tag{3}$$

* "Criteria for nodes in dupin's cyclides," *Ann. of Math.*, vol. 11, No. 5, p. 137 ff.

† *Bulletin Amer. Math. Soc.*, vol. 3, No. 7, p. 247 ff.