NOTE ON THE INVARIANTS OF n POINTS.

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A FAMILY of n points in ordinary space has $3n - t$ independent invariants by a t parameter Lie group. If the family is known to have p invariants there are then

$$q = p - 3n + t$$

relations among these p invariants. In particular if the group be the six parameter group of Euclidean motions,

$$p \quad q \quad r \quad yp - xq \quad zq - yr \quad xr - zp \tag{1}$$

where $p = \frac{\partial f}{\partial x}$, $q = \frac{\partial f}{\partial y}$, $r = \frac{\partial f}{\partial z}$, a system of n points has $3n-6$ independent invariants; but obviously the $\frac{n(n-1)}{2}$ mutual distances given by

$$\delta_{ij} \equiv ij = S(x_i - x_j)^2 \qquad i \neq j = 1, 2, \cdots, n. \tag{2}$$

are invariant by the group of motions; hence there are

$$q = \frac{n(n-1)}{2} - (3n - 6) = \frac{(n-3)(n-4)}{2}$$

relations among the δ_{ij}.

The invariants of the system of n points are found by the integration of the complete system of simultaneous partial differential equations

$$\sum_1^n{}^i \frac{\partial \varphi}{\partial x_i} = 0, \quad \sum_1^n{}^i \frac{\partial \varphi}{\partial y_i} = 0, \quad \sum_1^n{}^i \frac{\partial \varphi}{\partial z_i} = 0,$$

$$\sum_1^n{}^i \left(y_i \frac{\partial \varphi}{\partial x_i} - x_i \frac{\partial \varphi}{\partial y_i}\right) = 0, \quad \sum_1^n{}^i \left(z_i \frac{\partial \varphi}{\partial y_i} - y_i \frac{\partial \varphi}{\partial z_i}\right) = 0, \tag{3}$$

$$\sum_1^n{}^i \left(x_i \frac{\partial \varphi}{\partial z_i} - z_i \frac{\partial \varphi}{\partial x_i}\right) = 0;$$

and this system has at least $3n - 6$ solutions.