THEOREM. If $F(\xi)$ be an $IQ[p^s, p^n]$ belonging to the class λ (not the principal), $F(\xi^{p^n} - \xi)$ decomposes into p^n $IQ[p^s, p^n]$ of the class $\lambda + 1$; but if the former belong to the principal class, the latter is simply an $IQ[p^{s+1}, p^n]$ of the first class.

PARIS, *April* 15, 1897.

ON A SOLUTION OF THE BIQUADRATIC WHICH COMBINES THE METHODS OF DESCARTES AND EULER.

BY DR. EMORY McCLINTOCK.

[Read at the May meeting of the Society, 1897.]

THE product of

$$x^2 - v^{\frac{1}{2}} x + \tfrac{1}{2}(p + v + qv^{-\frac{1}{2}}) = 0 \qquad (1)$$

and

$$x^2 + v^{\frac{1}{2}} x + \tfrac{1}{2}(p + v - qv^{-\frac{1}{2}}) = 0 \qquad (2)$$

is

$$x^4 + px^2 + qx + \tfrac{1}{4}[(p+v)^2 - q^2v^{-1}] = 0. \qquad (3)$$

All of this except one term coincides with the short form of the general biquadratic,

$$x^4 + px^2 + qx + r = 0. \qquad (4)$$

Since v is at our disposal we may treat (3) and (4) as equivalent, term by term, so that we have, after clearing ing of fractions,

$$4rv = v(p + v^2) - q^2,$$

$$v^3 + 2pv^2 + (p^2 - 4r)v - q^2 = 0.* \qquad (5)$$

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* Up to this point this solution is precisely that of Descartes, except that the indeterminate quantity is here introduced in the form of a square root. It seems remarkable that the extreme facility with which the method of Descartes, which consists in separating the biquadratic into quadratic factors, may be combined with that of Euler, which consists in exhibiting the roots of the biquadratic as sums of square roots of the three roots of a cubic, should not heretofore have been observed. If the combination should, contrary to the writer's expectation, be found lacking in novelty, it may nevertheless be held that it has not attracted the attention which it deserves.