ON MODULAR EQUATIONS.

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The theory of the equations of transformation has been put in an entirely new light and very essentially improved by H. Weber's paper "Zur Theorie der Elliptischen Functionen.* His starting point is the solution of the equation for the division of the periods making a systematic use of the Galoisian theory of equations. From this standpoint it is not the modular equation

$$M(v, u) = 0$$

with coefficients rational in $u = \sqrt[4]{k}$ that we are led to consider but the equation

$$T(y, \varkappa) = 0$$

whose coefficients are rational in $\varkappa = k^2$ and whose roots are the $n + 1$ values of

$$\prod_{p=1}^{m} \frac{\text{cn}}{\text{dn}} \left(p \frac{4\lambda K + 4\mu i K'}{n} \right) \qquad \begin{array}{l} \lambda, \mu = 0, 1 \cdots n - 1 \\ \lambda = \mu = 0 \text{ excluded} \end{array}$$

Here for simplicity we take $n = 2m + 1$, an odd prime.

Let us see how the coefficients of this equation can be calculated. As the roots of T differ from those of M only by the factor u^{-n}, the T equations could be derived from the modular equations $M = 0$ on setting $v = u^n y$. But the methods given to compute the modular equations compel us—as far as I have been able to consult the literature—to pass from our domain of rationality $R(\varkappa)$ to that of $R(u)$, and this from our standpoint is certainly objectionable unless necessary. To show that this is not so is the *first object* of the present paper. Again, the methods given to calculate $M = 0$ are made—as far as I know—to depend upon the transformation theory of Hermite's function $u = \varphi(\tau)$. I propose to show as *second object* that we can calculate our T equation without leaving the ϑ's. A considerable simplification is thus obtained. But, having simplified the calculation of T so far, I have been tempted to go one step farther and show how we may arrive at Weber's equations

Acta Mathematica, vol. 6, p. 329. Also his book Elliptische Functionen und Algebraische Zahlen, Braunschweig, 1891.