TRANSCENDENTAL NUMBERS.

An authorized translation by Professor W. W. BEMAN of Chapter XXV. of Vol. II., of the *Lehrbuch der Algebra**

BY PROFESSOR HEINRICH WEBER.

[Professor Weber's presentation of the recent methods of demonstrating the transcendency of e and π, especially in sections 205 and 206, is so elementary that its reproduction in English will be welcomed by many. For the sake of completeness the whole chapter has been given. W. W. B.]

§ 203.

Enumerable Masses.

In the introduction to our work the general number concept was defined and with this general number concept we have operated, *e. g.*, in the proof of the existence of a root. In the further course of our investigations, we have dealt only with *algebraic numbers* without stopping to inquire whether the content of the number domain was thus exhausted, or there were non-algebraic numbers besides. The existence of non-algebraic numbers, also called transcendental, was first demonstrated by Liouville. Other proofs have been devised by G. Cantor.†

We begin here with the conception, already established in the introduction, of a *mass* or *manifoldness*, which means a system of elements of any kind so far defined that in case of any arbitrary object it is possible to decide whether it belongs to the system or not.

We distinguish between finite and infinite masses and introduce as our first and most important example of an infinite mass the aggregate of the *natural numbers* 1, 2, 3, . . . The following definition then holds :

1. *Definition. A mass is said to be enumerable* [*abzählbar*] *when its elements can be brought into a* (1, 1) *correspondence with the whole series of natural numbers or a portion of the same.*‡

Accordingly every finite mass is enumerable and in the enumeration the series of natural numbers is used only up to a certain greatest number. In what follows we specially consider infinite masses.

---

* Braunschweig, Vieweg und Sohn, 1895 und 1896.

† G. CANTOR, *Crelle's Journal*, vol. 77 (1873). "Ueber eine Eigenschaft des Inbegriffes aller reellen algebraischen Zahlen."

‡ CANTOR, *l. c.* The notion of enumerable masses agrees with the notion of *simply infinite systems* defined by DEDEKIND *without the assumption of the number system.* DEDEKIND, "Was sind und was sollen die Zahlen?" § 6. Braunschweig, 1887. (Second unaltered edition, 1893.)