ON THE STABILITY OF A SLEEPING TOP.

Abstract of a Lecture before the American Mathematical Society at the Princeton Meeting, October 17, 1896.

BY PROFESSOR FELIX KLEIN.

IN the four lectures* of the earlier part of the week I have attempted to simplify the formulæ for the motion of a top by turning to account the methods of the modern theory of functions. In treating this problem I have been largely influenced by the consideration that it is desirable on both sides to reinforce the relationships between pure mathematics and mechanics.

To-day I consider from the same standpoint a much more elementary question, which, however, for this very reason serves as a type for many related problems, viz., the stability of a top rotating about an axis directed vertically upward. The point of support we will assume to be fixed. If it were moveable in a horizontal plane, the formulæ would be somewhat more complicated, but the final result would be quite similar to that in the special case.

When the rotation is very rapid the behavior of the top is as if its axis were held fixed by a special force. This idea was employed, for instance, by Foucault (1851); to regard it, however, as an independent mechanical principle, as is done in many presentations of the subject, is, of course, absurd.

The usual mode of attacking the problem is by means of the *method of small oscillations*. If x, y are the horizontal coördinates of the point of support of the top, n its rotational velocity, and P the moment of its weight, then, rejecting higher powers of x and y, we obtain the linear homogeneous differential equations with constant coefficients

$$x'' + ny' - Px = 0,$$
$$y'' - nx' - Py = 0.$$

The terms in x' and y' in these equations are known as the gyroscopic terms. The solutions of the equations involve the characteristic exponent

$$\lambda = \frac{\pm in \pm \sqrt{4P - n^2}}{2}.$$

* Four lectures "On the theory of the top," delivered at the invitation of Princeton University in connection with its sesquicentennial celebration.