ON A REMARKABLE COVARIANT OF A SYSTEM OF QUANTICS.

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I wish to call attention in this paper to a remarkable covariant of a system of n quantics in n homogeneous variables. This covariant although not entirely new has not received the attention which it deserves. While the exposition here given is limited to the case of three quantics in three homogeneous variables, it will be seen that the methods are applicable to a system of n quantics in n variables. This covariant is here approached from the geometric point of view and the language used is that of the theory of higher plane curves. I shall attempt only to sketch the work in outline and shall omit all details as unsuitable to the place and object of this paper.

The covariant in question bears much the same relation to the Jacobian of the system of quantics as the Steinerian of a single quantic bears to the Hessian. In order therefore to get the most advantageous starting point for the discussion, we shall begin by restating two fundamental properties of the Jacobian.

Let there be given three curves, U, V and W, whose degrees are respectively m, n, and p. The Jacobian of these three curves, whose equation is obtained by equating to zero the functional determinant of U, V and W, has two well known polar properties as follows:

(1). *The Jacobian of U, V and W is the locus of all points common to the first polars of a point (x', y', z') with respect to U, V and W.*

(2). *The Jacobian of U, V and W is also the locus of the point (x', y', z'), whose polar lines with respect to U, V and W, meet in a point.*

The equation of the Jacobian is obtained in two different ways, each of which leads to one of the properties in question. The first polars of (x', y', z') with respect to U, V and W, are given by the following equations, using Salmon's notation (see Higher Plane Curves, Art. 61).

$$(1)\qquad \begin{aligned} x'U_1+y'U_2+z'U_3&=0,\\ x'V_1+y'V_2+z'V_3&=0,\\ x'W_1+y'W_2+z'W_3&=0. \end{aligned}$$

When these three equations are simultaneous their resultant vanishes, and we have the equation of the Jacobian and the proof of property (1).