In any case where Lindstedt series are applicable there are no asymptotic solutions, and, where there are asymptotic solutions, Lindstedt's series would be illusory.

We owe much to M. Poincaré for having commenced the attack on this class of questions. But the mist which overhangs them is not altogether dispelled; there is room for further investigation.

KRONECKER'S LINEAR RELATION AMONG MINORS OF A SYMMETRIC DETERMINANT.

BY PROFESSOR HENRY S. WHITE.

AMONG the minors of any determinant there exist well-known identical relations; those of lowest order, the quadratic relations, being readily obtained by the expansion of a determinant in which at least one pair of rows or of columns are identical. If, however, the original determinant is symmetrical, there are identities of a lower order than the quadratic, the linear identities first formally noticed by Kronecker in 1882.* These linear relations, published with no hint as to the manner of their discovery, are suggestive of a certain formula in such constant use as to have become a commonplace in the transformations of the Theory of Invariants of linear substitutions. The latter formula, however, relates to products of two determinant-factors, while Kronecker's is linear; but the latter uses double indices for the constituents, and herein lies the resemblance. By virtue of the ordinary process of multiplication of two determinants, Kronecker's theorem is easily proved to be a consequence from the other identity. Both are equally general, hence it seems likely that the earlier may have been the source of the later. This theory I will develop inductively, using for the sake of brevity determinants of three rows, and obtaining a typical linear relation among three-rowed minors of a six-rowed symmetric determinant.

Form an array of three rows of six constituents each:

$$\begin{matrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ b_1 & b_2 & b_3 & b_4 & b_5 & b_6 \\ c_1 & c_2 & c_3 & c_4 & c_5 & c_6 \end{matrix}$$

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* *Sitzungsberichte der Berliner Akademie*, 1882, p. 824. See also C. Runge: Die linearen Relationen zwischen den verschiedenen Subdeterminanten symmetrischer Systeme. *Jour. für r. u. a. Math.*, vol. 93 (1882), pp. 319–327.