NOTE ON THE COMMON TANGENTS OF TWO SIMILAR CYCLOIDAL CURVES.

BY PROFESSOR F. MORLEY.

In the *Educational Times* for November, 1895, appears a problem by Professor Aiyar,* from which it appears that from among the common tangents of two similar cycloids (not merely *right* cycloids, but epi- and hypocycloids) we can so select n (where n is the class of the curves) that three of them determine the rest; and that these n common lines touch a conic. This implies that all the common tangents, — say there are μn of them, where μ is to be found, — break up into μ sets of n, and determine μ conics. The fact of the common tangents separating into sets is interesting and, I think, new; and I propose *to determine the configuration formed by the* μ *conics*, using a method explained in the *American Journal* (vols. 15 and 16), which avoids difficulties which we should meet if we worked solely with the well-known line-equation of the curve; namely,

$$\delta = c \cos m\omega.$$

Working with the hypocycloid, we write its equation

$$(p-q)x/c = pt^{-q} - qt^{p}, \tag{1}$$

where p, q are positive integers with no common factor, and $p > q$. Here t is a complex number of absolute value 1, say a "turn"; and x is therefore complex. The real constant c is the radius of the vertex circle; and $t = 1$ gives a vertex. The equation, in fact, maps the unit circle $|t| = 1$ into the hypocycloid. The equation conjugate to (1) is

$$(p-q)y/c = pt^{q} - qt^{-p},$$

and this and (1) are parametric equations of the curve.

The tangent at the point t of the curve is

$$xt^{q} + qt^{p} = c(1 + t^{p+q}). \tag{2}$$

* Professor RAMASWAMI AIYAR, M. A. — A variable epi- or hypocycloid X similar to a given one touches three given straight lines continuously. Show that (1) the locus of the centre of X is a straight line; (2) X also touches $(n-3)$ other fixed lines where n is the class of the curve; (3) all these n lines are tangents to a conic; and (4) the envelope of the *vertex-circle* of X is the same conic. [In particular, if a cycloid extended both ways touches three given straight lines continuously, it touches an infinite number of other lines, all of which are tangents to a parabola; the envelope of the vertex-line of the cycloid is the same parabola.]