ON THE LOCUS OF THE FOCI OF CONICS HAVING DOUBLE CONTACT WITH TWO FIXED CONICS.

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THE system of conics having double contact with two conics includes as special cases the two systems of conics passing through four fixed points and touching four fixed lines, respectively. These latter result from making the fixed conics break up into factors, in the first case, in the tangential sense, and in the second case directly. Thus the general case includes and combines together into one whole the properties of the two special cases. Now I propose to consider the locus of the foci of the general system. This must include as special cases the known loci in the case of conics through four fixed points and touching four fixed lines. The first is a curve of the sixth order, which, when the points are concyclic, as has been shown by Professor Sylvester, breaks up into the two circular cubics having the points for foci, and the second is a circular cubic having its double focus on itself and passing through all the intersections of the four lines.

Now foci, being the intersections of tangents drawn to the curve from the imaginary points at infinity, are most easily treated by tangential coördinates, which I consequently use, the equations being precisely the same as the direct ones; viz. as given by Salmon, a conic having double contact with S, S' is, if $S - kS' = EF$;

$$\theta^2 E^2 - 2\,\theta\,(S + kS') + F^2 = 0.$$

But this being now a tangential equation, we may put

$$E = \lambda x_1 + \mu y_1 + \nu,$$

$$F = \lambda x_2 + \mu y_2 + \nu,$$

$$S + kS' = A\lambda^2 + B\mu^2 + C\nu^2 + 2\,F\mu^2 + 2\,G\nu\lambda + 2\,H\lambda\mu,$$

where x, y are rectangular Cartesian coördinates. Now the foci are the points of intersection of the tangents drawn parallel to $x \pm iy$. Hence for the foci we may put $\nu = x \pm iy$, $\lambda = -1$, $\mu = \mp i$; we thus get

$$\theta^2 (u - a)^2 - 2\,C\theta\,(u - c)\,(u - d) + (u - b)^2 = 0,$$

$$\theta^2 (v - a')^2 - 2\,C\theta\,(v - c')\,(v - d') + (v - b')^2 = 0, \qquad (1)$$

where $u = x + iy$, $v = x - iy$, and a, b, c, d are values of u for certain points, E, F, and two others, the corresponding values