than $n-1$, while the average number of elements in all the substitutions of $G$ is $n-1$. This is impossible, as the substitutions of $G$ which are not also in $H$ cannot contain more than $n-1$ elements. *Hence $H$ is transitive.* Since the average number of elements in the substitutions of $G$ is the same as that in the substitutions of $H$, *the substitutions of $G$ which are not found in $H$ must all be of the $(n-1)$th degree.*

The theorems of §§ 75, 76 of Netto's work * are proved by the preceding paragraph. It may be well to add in regard to the given § 75 that $G$ may be transitive while the corresponding subgroup of $G$ is intransitive. The following group † is an instance:

| | | | |
|---|---|---|---|
| 1 | 1 | $AB.CE$ | $af.bj.ci.dh.eg$ |
| $ABCDE$ | $abcde.fghij$ | $AC.DE$ | $ag.bf.cj.di.eh$ |
| $ACEBD$ | $acebd.fhjgi$ | $AD.BC$ | $ah.bg.cf.dj.ei$ |
| $ADBEC$ | $adbec.figjh$ | $AE.BD$ | $ai.bh.cg.df.ej$ |
| $AEDCB$ | $aedcb.fjihg$ | $BE.CD$ | $aj.bi.ch.dg.ef$ |

Netto's statement: "If $G$ is transitive in the $A$'s, $H$ is transitive in the $x$'s," together with the rest of the section seems to me to imply that such a case is impossible.

LEIPZIG, *August*, 1895.

# ON AN UNDEMONSTRATED THEOREM OF THE DISQUISITIONES ARITHMETICÆ.

BY DR. JAMES PIERPONT.

THE last section of the Disquisitiones Arithmeticæ contains the algebraic solution of the equations upon which the prime roots of unity depend.

$$x^p - 1 = 0, \ddagger \tag{1}$$

where $p$ is a prime. As this equation contains the factor $(x-1)$, we may consider instead the equation

$$x^{p-1} + x^{p-2} + \cdots + x + 1 = 0. \tag{2}$$

* COLE's translation, pp. 86, 87.

† COLE, *Quarterly Journal*, vol. 27, p. 41.

‡ The algebraic solution of these equations so simple in form presented difficulties which the mathematicians of the last century were not able to surmount. When $p = 11$ one arrives at an equation of 5th degree. Vandermonde gave the solution of this equation at the close of his paper Mémoire sur la Résolution des Equations, Hist. Acad. de Paris, 1771, but it appears to have been little known.