

The length of the arc of an asymptotic curve is given by the integral

$$s = \sqrt{\lambda} \int_v^\lambda \frac{dv}{\sqrt{(\lambda - v)(1 + \lambda - v)(1 - \lambda + v)}}.$$

Introducing the \wp -function with the invariants $g_2 = \frac{1}{4\lambda^2}$, $g_3 = 0$, and $e_1 = \frac{1}{4\lambda}$, $e_2 = 0$, $e_3 = -\frac{1}{4\lambda}$, ($k^2 = \frac{e_2 - e_3}{e_1 - e_3} = \frac{1}{2}$), we obtain:

$$\lambda - v = \frac{1}{4\lambda} \cdot \frac{1}{\wp s}.$$

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ON A GENERALIZATION OF WEIERSTRASS'S EQUATION WITH THREE TERMS.

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THE expression

$$\prod_{\lambda=1}^n \sigma(u - b_\lambda) / \sigma(u - a_\lambda)$$

is an elliptic function of u if

$$\sum a_\lambda = \sum b_\lambda.$$

The sum of the residues is zero; that is,

$$(1) \quad \sum_{\lambda=1}^n \frac{\sigma(a_\lambda - b_1) \dots \sigma(a_\lambda - b_{\lambda-1}) \dots \sigma(a_\lambda - b_{\lambda+1}) \dots \sigma(a_\lambda - b_n)}{\sigma(a_\lambda - a_1) \dots 1 \dots \sigma(a_\lambda - a_n)} = 0.$$

Being now only concerned with differences, we can, by a suitable addition to each a and b , write

$$(2) \quad \sum a_\lambda = \sum b_\lambda = 0.$$

When $n = 2$, the equation (1) is in no way characteristic of the σ -function, but is true of any odd function.

When $n = 3$, (1) becomes

$$(3) \quad \begin{aligned} & \sigma(a_1 - b_1) \sigma(a_1 - b_2) \sigma(a_1 - b_3) \sigma(a_2 - a_3) \\ & + \sigma(a_2 - b_1) \sigma(a_2 - b_2) \sigma(a_2 - b_3) \sigma(a_3 - a_1) \\ & + \sigma(a_3 - b_1) \sigma(a_3 - b_2) \sigma(a_3 - b_3) \sigma(a_1 - a_2) = 0. \end{aligned}$$