CONTINUOUS GROUPS.

SOPHUS LIE: *Vorlesungen über continuirliche Gruppen mit geometrischen und anderen Anwendungen.* Bearbeitet und ausgegeben von Dr. GEORG SCHEFFERS. Leipzig, Teubner, 1893. 8vo, pp. xi + 810.

IN the theory of transformation-groups we owe to Professor Lie a most important and interesting department of modern mathematics. The importance of the group idea itself has long been recognized in its application to the theory of substitutions, and some continuous transformations, such as the pedal transformation, were in use before Lie's work, but were used without their connection with the group idea being discovered, and this discovery and the presentation of the results of it in a systematic form are due to Professor Lie.

As far as the finite transformation-groups are concerned, these results have been gathered from the various journals in which they first appeared, and are available in a complete analytical form in the "Theory of Transformation Groups." *

In this book Professor Engel has accomplished with remarkable success the difficult task of presenting the general theory in a rigorous and clear manner. This general treatment, however, involves no small amount of difficulty, and to furnish an introduction to it the "Lectures on Continuous Groups" was written. At the same time it was aimed in this book alone to give, in outline at least, the general theory and indicate some lines in which it may be applied. Both of these ends Dr. Scheffers has successfully accomplished. The first part takes the reader through a discussion of transformation-groups in one and two variables, devoting especial attention to the projective groups and their most important subgroups. All the necessary principles of the projective geometry are derived as used, and a very elementary knowledge of mathematics is sufficient to enable the student to read this portion of the book. In the second part the general theory of continuous groups in n variables is dealt with. It is shown that if

$$x_i' = f_i(x_1 \ldots x_n a_1 \ldots a_r), \; i = 1 . 2 \ldots n$$

are the equations defining a continuous group, that there exist $r.n$ relations

$$\frac{\partial x_i'}{\partial a_k} = \sum_1^r {}_j \psi_{jk}(a_1 \ldots a_r) \xi_{ji}(x_1 \ldots x_n) \begin{cases} i = 1 . 2 \ldots n \\ k = 1 . 2 \ldots r \end{cases}$$

* Reviewed by Dr. CHAPMAN in the *Bulletin of the New York Mathematical Society*, vol. 2, No. 4.