KINETIC STABILITY OF CENTRAL ORBITS.*

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1. IN the chapter on Central Orbits of Tait and Steel's Dynamics of a Particle, Fourth Edition, p. 125, occurs an investigation of the apsidal angle of a nearly circular orbit. When the attraction varies inversely as the nth power of the distance, the expression found becomes imaginary when n exceeds 3, and the remark is made that "the investigation furnishes a simple example of the determination of the conditions of *Kinetic Stability*, which we cannot discuss in this elementary treatise." It may not be without interest to show that an investigation of a no less elementary character will furnish a satisfactory discussion of this interesting subject, so far as it relates to central forces.

2. The usual polar equation of the central orbit is

$$\frac{d^2u}{d\theta^2} + u = \frac{P}{h^2u^2}, \tag{1}$$

in which P is the attraction acting on a unit of mass.

The first integral of this equation found in the usual way is

$$\left(\frac{du}{d\theta}\right)^2 + u^2 = \frac{2}{h^2}\int\frac{Pdu}{u^2}.$$

Since $\int\frac{Pdu}{u^2} = -\int Pdr = -V + C$, V being the potential function, this equation may be written

$$\left(\frac{du}{d\theta}\right)^2 = \frac{2}{h^2}(C - V) - u^2 = \psi(u), \tag{2}$$

the function $\psi(u)$ depending not only on the given law of force, but also upon the values given to the two constants h and C.

3. In discussing the function $\psi(u)$ we have only to consider positive values of u, (the reciprocal of r the distance from the centre of force,) and it is evident from equation (2) that the values of u for every point of an actual orbit must be such as to make $\psi(u)$ positive, except the maxima and minima values

* Read before the AMERICAN MATHEMATICAL SOCIETY at the meeting of April 27, 1895.