GERGONNE'S PILE PROBLEM.

BY MR. LEONARD E. DICKSON.

IN Ball's *Mathematical Recreations*, pp. 101–6, is described the familiar three-pile trick with twenty-seven cards, mentioned by Bachet; also Gergonne's generalization for a pack of m^m cards. Suppose the pack is dealt into m piles of m^{m-1} cards each, and after the first deal the pile indicated as containing the selected card is taken up ath; after the second deal is taken up bth, and so on; finally, after the mth deal is taken up kth. Then the card selected will be the nth from the top where,

$$\text{if } m \text{ is even,} \quad n = km^{m-1} - jm^{m-2} + \dots + bm - a + 1;$$
$$\text{if } m \text{ is odd,} \quad n = km^{m-1} - jm^{m-2} + \dots - bm + a.$$

If in the latter case we put the pile indicated always in the middle of the pack, then $a = b = \dots = j = k = \dfrac{m+1}{2}$, and we find $n = \dfrac{m^m + 1}{2}$, or the card will appear in the middle of the pack.

Dr. C. T. Hudson * has discussed the general problem: To deal a pack of ab cards into a piles of b cards each and so stack the piles after each deal that after the nth deal any selected card may be rth in the whole pack. Let the card selected be the sth card from the bottom of the pile containing it after the first deal. Let $p_1, p_2, \dots p_n$ be the places the pile of the selected card is to hold after the first, second, ... nth stacking of the piles. After the first stacking the number of the selected card, counting from the bottom of the pack, will be the $b(p_1 - 1) + s$; after the second, $b(p_2 - 1) + \dfrac{b(p_1 - 1) + s + m_1}{a}$; after the nth,

$$\frac{1}{a^{n-1}}[b\{a^{n-1}(p_n - 1) + a^{n-2}(p_{n-1} - 1) + \dots + a(p_2 - 1) + (p_1 - 1)\} + s + m_1 + am_2 + \dots + a^{n-2}m_{n-1}],$$

where $m_1, m_2, \dots m_{n-1}$ are integers $\overline{\underline{<}}\, a - 1$, including 0. Since $m_1 + am_2 + \dots + a^{n-2}m_{n-1}$ lies between 0 and $a^{n-1} - 1$, the selected card's place r lies between

$$\frac{1}{a^{n-1}}[b\{a^{n-1}(p_n - 1) + \dots + a(p_2 - 1) + (p_1 - 1)\} + s]$$

* *Educational Times Reprints*, 1868, vol. 9, pp. 89–91.