tribute to the development of science, the really new impulses can be traced back to but a small number of eminent men. But the work of these men is by no means confined to the short span of their life; their influence continues to grow in proportion as their ideas become better understood in the course of time. This is certainly the case with Riemann. For this reason you must consider my remarks not as the description of a past epoch, whose memory we cherish with a feeling of veneration, but as the picture of live issues which are still at work in the mathematics of our time.

THE MULTIPLICATION OF SEMI-CONVERGENT SERIES.

BY PROFESSOR FLORIAN CAJORI.

In *Math. Annalen*, vol. 21, pp. 327-378, A. Pringsheim developed sufficient conditions for the convergence of the product of two semi-convergent series, formed by Cauchy's multiplication rule, when one of the series becomes absolutely convergent, if its terms are associated into groups with a finite number of terms in each group. The necessary and sufficient conditions for convergence were obtained by A. Voss (*Math. Annalen*, vol. 24, pp. 42–47) in case that there are two terms in each group, and by the writer (*Am. Jour. Math.*, vol. 15, pp. 339–343) in case that there are p terms in each group, p being some finite integer. In this paper it is proposed to deduce the necessary and sufficient conditions in the more general case when the number of terms in the various groups is not necessarily the same.

Let $U_n = \sum_0^n a_n$ and $V_n = \sum_0^n b_n$ be two semi-convergent series, and let the first become absolutely convergent when its terms are associated into groups with some finite number of terms in each group. Let r_n represent the number of terms in the $(n+1)$th group, and let g_n represent the $(n+1)$th group embracing r_n terms. Let, moreover, a_{R_n} represent the first term in the group g_n, where $R_0 = 0$ and $R_n = r_0 + r_1 + r_2 + \ldots + r_{n-1}$, then

$$g_n = (a_{R_n} + a_{R_n+1} + a_{R_n+2} + \ldots + a_{R_n+r_n-1}) \quad \text{and} \quad U_n = \sum_0^n g_n.$$

Since, by a theorem of Mertens, the product of an absolutely convergent series and a semi-convergent series, formed