ON THE INTRODUCTION OF THE NOTION OF HYPERBOLIC FUNCTIONS.*

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THE difficulties in the way of a satisfactory geometrical deduction of the fundamental formulæ of the hyperbolic functions seem to be due to the lack of a definition of these functions which shall be independent of the particular position of the argument area. A general definition of this kind can, however, readily be found in terms of the ratios of certain *areas*, instead of *lines*. From this definition the addition-theorem and other characteristics can be easily deduced by the methods of analytic geometry; and the definitions hold, furthermore, not merely for the rectangular, but for *any* hyperbola.

I. *The circular functions.* In order to bring out clearly the analogy with the circular functions, I will first indicate briefly how the latter would be defined according to this method.

In a circle of radius a (Fig. 1) let ϕ be the angle between the radii OP and OQ, and let OP' be drawn perpendicular to

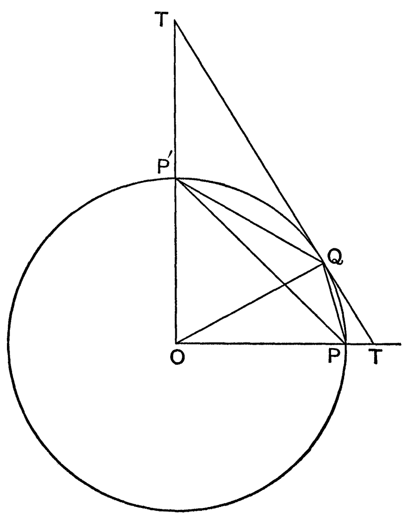

FIG. 1.

* Read before the AMERICAN MATHEMATICAL SOCIETY, December 28, 1894. For various geometrical definitions of these functions, see Professor A. Macfarlane's paper: "On the definition of the trigonometric functions," 1894.—EDITORS.