where λ, μ, ν, ρ are tangential co-ordinates. The ratios of the differences of $\alpha, \beta, \gamma, \delta$ give the two degrees of freedom which the curve still possesses. Now by eliminating t we may write down

$$\left.\begin{array}{l}(\beta-\gamma)\mu\nu+(\gamma-\alpha)\nu\lambda+(\alpha-\beta)\lambda\mu=0,\\ (\beta-\delta)\mu\rho+(\delta-\alpha)\rho\lambda+(\alpha-\beta)\lambda\mu=0,\\ (\gamma-\delta)\nu\rho+(\delta-\beta)\rho\mu+(\beta-\gamma)\mu\nu=0,\end{array}\right\} \qquad (10)$$

which are three tangential quadrics touching the osculating planes of the curve and not connected by a linear relation. Substituting, then, $\frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz}, \frac{d}{du}$ for λ, μ, ν, ρ, respectively, in these three expressions and operating on the quadric given above, we get

$$\begin{array}{l}(\beta-\gamma)f+(\gamma-\alpha)g+(\alpha-\beta)h=0,\\ (\beta-\delta)m+(\delta-\alpha)l+(\alpha-\beta)h=0,\\ (\gamma-\delta)n+(\delta-\beta)m+(\beta-\gamma)f=0,\end{array}$$

whence

$$(h-l)(g-f)(f-m)+(h-g)(m-h)(n-f) \\ +(h-f)(h-l)(n-f)=0, \qquad (11)$$

which is consequently the invariant relation connecting the cubic surface and the quadric when they are capable of being written in the forms (5). Thus it appears that a cubic surface and a quadric cannot, in general, be reduced to the forms (5), and that when they are reducible to these forms, the reduction can take place in a singly infinite number of ways, all the planes x_1, x_2, etc., involved being osculating planes of a given twisted cubic.

HAYWARD'S VECTOR ALGEBRA.

The Algebra of Coplanar Vectors and Trigonometry. By R. Baldwin Hayward. Macmillan & Co., 1892. 8vo, pp. xxix + 343.

It is a curious fact that while the English are the one nation which in elementary geometry clings to Euclid, the prototype of mathematical rigor, not only is most recent English mathematical work, however excellent in many respects, decidedly lacking in rigor of form, but many English mathematical writers of the present day show an entire lack of critical sense which if shown in elementary geometry would discredit a schoolboy.