

ON THE PROBLEM OF THE MINIMUM SUM OF  
THE DISTANCES OF A POINT FROM GIVEN  
POINTS.\*

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1. *Introductory Note.*—Although repeatedly treated before,† the problem of determining a point such that the sum of its distances from a number of given points shall be a minimum still offers room for discussion in two respects.

First, the question as to the best method of investigation cannot yet be regarded as settled. The ordinary method of the differential calculus not only leads to unnecessary complications owing to the use of co-ordinates not essentially connected with the problem, but also necessitates the exclusion of negative distances, a serious difficulty that does not seem to have been

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† See, for instance, the references to the literature of the problem given by STURM in *Crelle's Journal für Mathematik*, vol. 97, p. 51.