LOBACHÈVSKY AS ALGEBRAIST AND ANALYST.

BY PROF. A. VASILIEV.

THE mathematical genius of Lobachèvsky manifested itself not in geometry alone. His early study of Gauss's *Disquisitiones arithmeticæ*, under the direction of Professor Bartels, led him in 1813 to write a memoir (which has never been published and seems to be lost) on the resolution of the binomial equation $x^n - 1 = 0$ in the case $n = 4p + 1$. At a later period, in 1834, he returned to these studies and carefully examined the case $n = 8p + 1$ (see his paper "Reduction of the degree of the binomial equation when the exponent, diminished by 1, is divisible by 8," written in Russian and published in the first volume (1834) of the Scientific Memoirs of the University of Kazàn).

Next to geometry, Lobachèvsky's favorite subject was the systematic exposition of, algebra. He considered algebra as intended to lay the rigorous foundations for mathematical science, and this idea he carried out in a work published in 1833 under the title "Algebra, or calculus of finite quantities" (Russian). This extensive work is remarkable alike for the rigor of its definitions and proofs, and for the width of its scope. Thus we find here treated not only the solution of numerical equations and Gauss's theory of the resolution of the equation $x^n - 1 = 0$, but also the calculus of finite increments. Through his geometrical researches Lobachèvsky had been led to the necessity of defining the trigonometric functions independently of all geometric considerations ; this idea he introduced into his algebra, in which the trigonometric functions $\cos z$ and $\sin z$ are defined by means of the series $1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \ldots$ and $\frac{z}{1} - \frac{z^3}{3!} + \frac{z^5}{5!} - \ldots$, and all their properties are derived from this definition. I believe that Lobachèvsky was the first to expound systematically the theory of trigonometric functions on this basis.*

* Professor A. Macfarlane in his interesting paper "On the definitions of the trigonometric functions" mentions De Morgan as expressing the same idea in his "Trigonometry and Double Algebra." Professor Mansion attributes the idea to Mr. Seidel (*Crelle's Journal für Math.*, vol. 73, 1871).

[This must be a misunderstanding, as Seidel's paper deals with products and not with series. Cauchy, in his "Analyse algébrique," 1821, p. 309, distinctly defines $\sin z$ and $\cos z$ by means of the infinite series. —A. Z.]