ON THE GENERAL TERM IN THE REVERSION OF SERIES.

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IN reverting the series

$$y = a_0 x + a_1 x^2 + a_2 x^3 + \ldots \qquad [a_0 \neq 0]$$

it is usual to assume a development for x in the form

$$x = A_0 y + A_1 y^2 + A_2 y^3 + \ldots,$$

and then to substitute, and equate coefficients of like powers, thus determining $A_0, A_1, \ldots$ in succession.

This method does not give any observable law for the independent formation of the expression for the coefficient of a given power of y.

A different method, however, based on Lagrange's series, furnishes the desired general term.

The first equation may be written

$$a_0 x = y - a_1 x^2 - a_2 x^3 - \ldots,$$

or

$$x = z + b_1 x^2 + b_2 x^3 + \ldots, \quad = z + \phi(x),$$

where

$$z = \frac{y}{a_0}, \quad b_1 = -\frac{a_1}{a_0}, \quad b_2 = -\frac{a_2}{a_0}, \quad \ldots,$$

and

$$\phi(x) = b_1 x^2 + b_2 x^3 + \ldots;$$

whence, by Lagrange's series,

$$x = z + \phi(z) + \frac{1}{2!}\frac{d}{dz}[\phi(z)]^2 + \frac{1}{3!}\frac{d^2}{dz^2}[\phi(z)]^3 + \ldots$$