

THREE NOTES ON PERMUTATIONS.

BY PROFESSOR F. MORLEY.

1. *A plea for the chess-board in teaching determinants.*

Not a few writers on determinants, whether in works devoted to the subject or in an incidental treatment, dispose of the "rule of signs" with such a statement as: "The sign + or - is to be prefixed to a term according as it can be derived from the leading term by an even or odd number of interchanges of suffixes." It will, I think, be conceded that the rule, so stated, appears very arbitrary to a class; and I suppose that many teachers, like myself, have experienced "a certain gaspingness" in laying it down. Of course the difficulty disappears when the characters of even and odd permutations have been first explained, as is done, for example, by R. F. Scott in his treatise. The chief question for the teacher then is: which rule for determining the nature of a permutation shall I recommend—Cramer's or Cauchy's? And the question seems answered by the terms in which Jacobi (*Crelle*, vol. 22, p. 287) refers to these rules after proving them; he refers to Cramer's rule as "quam regulam," and to Cauchy's as "hanc pulchram regulam." See also the remarks of Muir, *History of the Theory of Determinants*, vol. 1, p. 247.

As the basis of this first note, Cauchy's writings, in particular the memoir which appeared in the *Comptes Rendus*, vol. 12, March 1841 (*Works*, series 1, vol. 6, p. 87; summarized in Muir's *History*, vol. 1, p. 234), must be especially referred to. I do not think that it has been sufficiently pointed out how to render the ideas of his abstract presentation intelligible to the average beginner, and I venture, in this first note, to sketch what seems to me the best way of presenting the whole matter to a class, namely, by fusing the idea of a term of a determinant with that of a permutation. I omit a proof which will be found in the section on substitutions in Serret's *Algèbre Supérieure*, or in Muir; the changes to be made being merely verbal.

We are not concerned with the size of the constituents, but only with certain elementary questions of permutations.* Only, instead of regarding the things to be permuted as restricted to a line, we regard them as material "men" actually placed on a material chess-board, in the first or standard case

* It is perhaps inadvisable to say much about substitutions in this connexion.