LACHLAN'S MODERN PURE GEOMETRY.

An Elementary Treatise on Modern Pure Geometry. By R. LACHLAN, M.A. Macmillan & Co., 1893. 8vo, pp. x. + 288.

IN this work, with slight exception, the field of operations is the plane. In the plane there are considered points, lines, and circles; and concerning these there is an excellent store of geometric facts, illustrating various methods. Of these methods so many are to be found in English works on plane geometry, whether they are called pure or analytic, that the delimitation is not very evident. It appears to be accepted as the fundamental distinction that in the books on pure geometry the treatment of curves by means of their equations is excluded. Whether co-ordinates are excluded is mainly a question of words; the algebra employed does not look like algebra, but this is of no serious importance; what is really out of bounds is the equation between variable co-ordinates, or the theory of the ternary form. With respect to the use of algebra, it can hardly be maintained, in the present case, that every equation ought to be associated in one's mind with a direct geometric concept; this would, in many cases, be a direct waste of energy, as, for instance, where trigonometry is employed. Since some algebra is permitted, the question arises whether a formal use of the algebra of binary forms is not proper in a work of this kind, and I shall briefly return to this point later.

In Dr. Lachlan's book, after three preliminary chapters, we have the explanation of harmonic ranges and pencils, and then a chapter on involution. The statement (page 40) that the double points of an involution can be imaginary, is misleading; it is better to say that they can be off the line in question. Chapter VI. gives an account of the triangle and of some of its more remarkable circles. Then follows (chapter VII.) the theory of four points or lines, or, as they are called after Townsend, tetrastigms and tetragrams. One hopes that more pleasing pet names will yet be adopted. This chapter contains some extensions which deal mostly with six points or lines. Among these may be noticed (page 99) the case of two triangles triply in perspective, or rather of three triangles, any two of which are in perspective as seen from any point of the third. A simple instance is when two equilateral triangles have the same centre; the centres of perspective form a third equilateral triangle concentric with the others.

The theory of perspective is contained in chapter VIII. The symmetry of the diagram on page 100 should be re-