THE TRANSITIVE SUBSTITUTION-GROUPS OF NINE LETTERS.

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The list of these groups published by Mr. Askwith in the present volume (26) of the Quarterly Journal of Mathematics contains only 22 of the 34 actually existing types. The groups identified by Mr. Askwith are the more obvious forms—the symmetric and alternating groups and the non-primitives—together with two primitive types of order 72, while the omitted cases—5 non-primitives and 7 primitives—include among others such specially interesting forms as the triad group of order 216 identified with the theory of the points of inflection of the plane cubic curve, etc., and a *simple* group, apparently heretofore unrecognized, of order 504.

I give here a complete list of the transitive groups of this degree, together with very brief explanations of the processes by which I have obtained them.

A. *The Non-primitive Groups of Nine Letters.*

In respect to these groups the nine letters are distributed in three systems of three letters each, such that every substitution of the group replaces every system either by itself or by one of the other systems.

Those substitutions of the group which replace every system by itself form a self-conjugate subgroup, i.e., a subgroup which is transformed into itself by every substitution of the group. This subgroup obviously affects the three systems symmetrically.

Of the intransitive groups of nine letters with three transitive systems of three letters each only the following 8 satisfy the last requirement :

$H_{216} =$ (abc) all (def) all (ghi) all,
$H_{108} =$ $\{(abc)$ all (def) all (ghi) all$\}$ pos.,
$H_{54} =$ (abc) pos. (def) pos. (ghi) pos. $+$
(abc) neg. (def) neg. (ghi) neg.,
$H_{27} =$ (abc) pos. (def) pos. (ghi) pos.,

$H_{18} =$

1*,	1,	$abc \cdot def$,	$acb \cdot dfe$,
ghi,	abc,	$acb \cdot def$,	dfe,
gih,	acb,	$abc \cdot dfe$,	def,
gh,	$ab \cdot de$,	$ac \cdot df$,	$bc \cdot ef$,
gi,	$ac \cdot de$,	$bc \cdot df$,	$ab \cdot ef$,
hi,	$bc \cdot de$,	$ab \cdot df$,	$ac \cdot ef$,

* I use this notation to indicate that every substitution of e, f, g is multiplied by the 3 several substitutions opposite it.