

A COMPARATIVE REVIEW OF RECENT RE-
SEARCHES IN GEOMETRY.*

(PROGRAMME ON ENTERING THE PHILOSOPHICAL FACULTY AND
THE SENATE OF THE UNIVERSITY OF ERLANGEN IN 1872.)

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Prefatory Note by the Author.—My 1872 Programme, appearing as a separate publication (Erlangen, A. Deichert), had but a limited circulation at first. With this I could be satisfied more easily, as the views developed in the Programme could not be expected at first to receive much attention. But now that the general development of mathematics has taken, in the meanwhile, the direction corresponding precisely to these views, and particularly since *Lie* has begun the publication in extended form of his *Theorie der Transformationsgruppen* (Leipzig, Teubner, vol. I. 1888, vol. II. 1890), it seems proper to give a wider circulation to the expositions in my Programme. An Italian translation by M. Gino Fano was recently published in the *Annali di Matematica*, ser. 2, vol. 17. A kind reception for the English translation, for which I am much indebted to Mr. Haskell, is likewise desired.

The translation is an absolutely literal one; in the two or three places where a few words are changed, the new phrases are enclosed in square brackets []. In the same way are indicated a number of additional footnotes which it seemed desirable to append, most of them having already appeared in the Italian translation.—F. KLEIN.

Among the advances of the last fifty years in the field of geometry, the development of *projective geometry* † occupies the first place. Although it seemed at first as if the so-called metrical relations were not accessible to this treatment, as they do not remain unchanged by projection, we have nevertheless learned recently to regard them also from the projective point of view, so that the projective method now embraces the whole of geometry. But metrical properties are then to be regarded no longer as characteristics of the geometrical figures *per se*, but as their relations to a fundamental configuration, the imaginary circle at infinity common to all spheres.

When we compare the conception of geometrical figures gradually obtained in this way with the notions of ordinary (elementary) geometry, we are led to look for a general principle in accordance with which the development of both methods has been possible. This question seems the more important as, beside the elementary and the projective geometry, are arrayed a series of other methods, which, albeit they are less developed, must be allowed the same right to an individual existence. Such are the geometry of reciprocal radii

* Translated by Dr. M. W. HASKELL, Assistant Professor of Mathematics in the University of California.

† See Note I. of the appendix.