A CASE OF NON-EUCLIDIAN GEOMETRY.*

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THE note here presented was suggested by the very interesting article by Dr. McClintock in the November number (vol. II., p. 21) of the BULLETIN, and merely presents in another point of view the pseudo-measurements, occurring in the Cayley interpretation in the case in which c is positive, developed by Dr. McClintock (page 27 *et seq.*) under the name of *projective distances* as distinguished from the real distance of Euclidian geometry.

In the theory of projective metrics, the case is the symmetric one of the opposite variety from that of Lobatschewsky, or that in which the absolute quadric of Cayley is an imaginary sphere.

In the explanation given in the BULLETIN it will be remembered that, for the geometry of the plane, we have only to assume a *central point* of the plane, and a sphere of fixed radius touching it at that point ; then for any finite straight line or *sect* in the plane (not simply a sect of a line passing through the central point) the *projective measure* is the length of the arc of a great circle which is the central projection of the sect upon the spherical surface. In like manner the *projective measure* of an angle is the spherical angle between the great circles which are the projections upon the sphere of the sides of the given angle. Thus a plane triangle is represented by a certain spherical triangle, and we develop a non-euclidian geometry of two dimensions, of which the theorems are those of spherical geometry.

What the non-euclidian and two-dimensional dwellers in the plane would call the same triangle in different positions would be such triangles as had for projections the same spherical triangle in different positions upon the sphere.

In extending these projective measures to geometry of three dimensions, we seek to establish, with reference to a single assumed *central point* in space, a consistent system of projective measurement for all lines and angles however situated in space. In doing this we shall establish for every plane in space its central point, and with reference to this point a system of projective measurement in the plane similar to but not identical with that described above. For this purpose Dr. McClintock distinguishes a *prime* plane in which the projective measurement is defined by means of a tangent sphere whose radius is r, and a *secondary* plane in

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