ON A GENERAL FORMULA FOR THE EXPANSION OF FUNCTIONS IN SERIES.*

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1. If O be the symbol which represents any operation performed on a function, and O^r the repetition of that operation r times, then the formula referred to above is

$$\begin{vmatrix} fx & f_1x & \ldots & f_nx & f_{n+1}x \\ fy_1 & f_1y_1 & \ldots & f_ny_1 & f_{n+1}y_1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ fy_p & f_1y_p & \ldots & f_ny_p & f_{n+1}y_p \\ Ofx_1 & Of_1x_1 & \ldots & Of_nx_1 & Of_{n+1}x_1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ O^qfx_q & O^qf_1x_q & \ldots & O^qf_nx_q & O^qf_{n+1}x_q \\ \Phi(u) & 0 & \ldots & 0 & 1 \end{vmatrix} \qquad (1)$$

in which all elements of the last row except the first and last are zero. The symbol O^rfx_t means that after the rth operation on fx, the argument is changed into x_t. $\Phi(u)$ represents, in general, some function of $x, y_1, \ldots, y_p, x_1, \ldots, x_q$, involving also the form of the functions in the determinant.

If now the operation O be such that the Φ function may be so determined that the above determinant vanishes, we have, regarding x as the variable, the formulæ

$$fx = A_1f_1x + \ldots + A_{n+1}f_{n+1}x,$$
$$fx = B_1fy_1 + \ldots + B_pfy_p$$
$$+ C_1Ofx_1 + \ldots + C_qO^qfx_q + D\Phi(u).$$

The first of these may be regarded as an expansion of fx according to the functions $f_1x, \ldots f_{n+1}x$, whose coefficients are independent of the argument x, save in so far as Φ is a function of x. The second, in turn, may be regarded as an expansion of fx according to the form fy_r and the successive operatives of fx, whose coefficients are independent of the form of the function fx; the residual term being $D\Phi(u)$, wherein D does not depend on the form of the function fx.

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