

## ON THE NON-EUCLIDIAN GEOMETRY.

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The celebrated tracts of Lobatschewsky and Bolyai, in which those writers showed what geometry might become if the parallel-axiom were left out, were long since translated into the chief languages of the continent, but have until the last year remained inaccessible to those whose only tongue is the English. The thanks of this large class are due to Professor Halsted \* for supplying the deficiency in good clear style. The critical remarks made by Professor Halsted upon geometrical text books in current use show much acuteness, but they partake of the nature of ephemeral polemics, and will doubtless be omitted in any future edition of these translations, which ought to be republished together in permanent form as a standard work. The practical services which he has rendered to other mathematicians, not only by his valuable text-books, but also in the publication of his elaborate "Bibliography of Hyper-Space and Non-Euclidian Geometry" (*American Journal of Mathematics*, vols. 1 and 2), and now by these translations, are eminently deserving of appreciation and imitation.

Lobatschewsky, acting on suggestions of Gauss, delivered his first lectures on non-euclidian geometry in 1826, his completed work following in 1840. Bolyai's tract was published in 1832 as an appendix to a work of his father, who was also a friend of Gauss. Both of these authors begin their investigations by assuming that through a given point in a plane more than one line can be drawn which shall never meet a given line. Their results, as far as they cover the same ground, are identical in substance, though different in form. The sum of the angles of a triangle is less than two right angles, so that a rectangle is impossible; the angle-sums of two triangles of equal area are equal; no two triangles not equal can have the same angles, so that similar triangles not of the same size are impossible; if two equal perpendiculars are erected to the same line, their distance apart increases with their length; a line every point of which is equally distant from a given straight line is a curved line; any two lines which do not meet, even at infinity, have one common perpen-

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\* Geometrical Researches on the Theory of Parallels. By Nicolaus Lobatschewsky. Berlin, 1840. Translated by George Bruce Halsted. Austin: published by the University of Texas, 1891.

*Scientiæ Baccalaureus*, vol. 1, No. 4, June, 1891. The Science Absolute of Space. By John Bolyai. Translated into English by George Bruce Halsted.