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EVOLUTION OF CRITERIA OF CONVERGENCE.

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THE expressions convergent and divergent series were used for the first time in 1668 by James Gregory. Newton and Leibniz felt the necessity of inquiring into the convergence of infinite series, but they had no proper criteria, excepting a test advanced by Leibniz for alternating series. By Euler and his contemporaries the *formal* treatment of series was greatly extended, while the necessity for determining the convergence was generally lost sight of. To be sure, it was Euler who first observed the semi-convergence of a series. He, moreover, remarked that great care should be exercised in the summation of divergent series. But this warning was not taken so seriously by him as it would be by a modern writer, for in the very same article* in which it occurs Euler did not hesitate to write

$$\ldots + \frac{1}{n^2} + \frac{1}{n} + 1 + n + n^2 + \ldots = 0,$$

simply because

$$n + n^2 + \ldots . = \frac{n}{1-n}; \; 1 + \frac{1}{n} + \frac{1}{n^2} + \ldots = \frac{n}{n-1}.$$

The facts are that Euler reached some very pretty results in infinite series, now well known, and also some very absurd results, now quite forgotten. Protests were made by Nicolaus Bernoulli and Varignon against the prevailing reckless use of series : isolated attempts at establishing criteria of convergence are on record : but the dominating sentiment of the age frowned down any proposition which would put limitations upon operations with series. The faults of this period found their culmination in the Combinatorial School in Germany, which has now passed into deserved oblivion.

I. *Special Criteria.* In the progress of mathematics, truth

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* *Comm. Petrop.*, vol. 11, p. 116.