COLLINEATION AS A MODE OF MOTION.*

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IN the following paper I have attempted to give an account of some very simple matters, which, although familiar to many, appear to have attracted but little attention in this country. The subject, however, has never, as far as I know, been presented from precisely the point of view here adopted.

Perhaps the most important difference between the old and the new geometry lies in the extended use made during the present century of geometric transformations.† The change which has come about in this direction is due in part to the influence of certain branches of applied mathematics in which one has to deal not merely with geometric configurations but also with certain changes which these configurations are forced to undergo. There are however two distinct ways of looking at a transformation. First we may consider the original and the transformed figure as standing side by side, or even as occupying portions of the same space, the latter being in a certain sense a picture of the former ; or secondly, we may consider the original figure to be gradually deformed according to a given law into the transformed figure. Each of these points of view can be traced to a physical origin. Perspective and allied subjects strikingly illustrate the first, while the second will most naturally be adopted in hydrodynamics, the theory of elasticity, etc. Now while the first of the above mentioned ways of looking at a transformation has the advantage of introducing no unnecessary element into the consideration, the second in turn has the advantage of making the idea of a transformation lose much of its abstractness, for by its aid we are enabled to see the points of the original figure rearrange themselves by a gradual motion into the transformed figure.

I wish to illustrate this way of looking at a transformation as a mode of motion by considering one of the simplest of transformations, the so-called linear transformation or collineation,‡ and for the sake of simplicity I will confine myself to two dimensions.

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† The following remarks should be understood to apply only to point transformations, *i.e.*, to transformations which carry points over into points.

‡ The word collineation seems to be by far the best name for this transformation, not only because it is as applicable in synthetic as in analytic geometry, but also because the ambiguity which arises in speaking of a