$4\nu + 1$, $4\nu + 3$. *When* $n = 4\nu + 2$, *no odd branch can exist. Exceptional are the cases when* $n = 3, 4, 5$, *the maximum number of odd branches being* 1, 2, 3, *respectively.* Then, by applying Abel's theorem for elliptic functions, he proves, for every value of n, the existence of curves with the maximum number of real odd branches.

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FINAL FORMULAS FOR THE ALGEBRAIC SOLUTION OF QUARTIC EQUATIONS.*

BY MANSFIELD MERRIMAN, PH.D.

I. FINAL formulas for the algebraic solution of quadratic and cubic equations are well known. Such formulas exhibit the roots in their true typical forms, and lead to ready and exact numerical solutions whenever the given equations do not fall under the irreducible case. But for the quartic, or biquadratic, equation the books on algebra do not give similar final formulas. The solution of the quartic has been known since 1540, and numerous methods have been deduced for its algebraic resolution, yet in no case does this appear to have been completed in final practical shape. It is the object of this paper to state the final solution in the form of definite formulas.

II. The expression of the roots of the quartic is easily made in terms of the roots of a resolvent cubic, and the cubic itself is solved without difficulty. Yet great practical difficulty exists in treating a numerical equation on account of the presence of imaginaries in the roots of the resolvent. Witness the following example which is generally given to illustrate the method in connection with Euler's resolvent:

"Let it be required to determine the roots of the biquadratic equation,

$$x^4 - 25x^2 + 60x - 36 = 0.$$

By comparing this with the general form the cubic equation to be resolved is,

$$y^3 - 50y^2 + 729y - 3600 = 0$$

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