# TOPOLOGY OF ALGEBRAIC CURVES.

In the *Mathematische Annalen*, Vol. 38 (1891), Mr. David Hilbert of Königsberg has a very interesting and suggestive article on the real branches of algebraic curves. The simplicity of the method which Mr. Hilbert employs, and the possibility of its being made to yield further important results seem sufficient reasons for presenting here, in some detail, that portion of the article which treats of plane curves. It has seemed to the present writer advisable to amplify portions of Mr. Hilbert's article, with the view of making his method more intelligible, and also to make some changes in the proof of the principal theorem, in order to avoid some slight inaccuracies that have crept into his demonstration.

The first part of the article in question is devoted to the determination of the maximum number of *nested branches* possible to a plane algebraic curve of order $n$, and of maximum deficiency. By *nested branches* is meant a group of even branches so arranged that the first lies entirely within the second, the second within the third, and so on, like a series of concentric circles.* It should be observed that some or all of the non-nested branches may, in perfect accord with this definition, lie within the ring-shaped regions formed by the nested branches. A single even branch, which neither encloses another branch nor is enclosed by one, may be looked upon as a nested branch or not, according to the nature of the question under discussion. For reasons that will presently appear, Hilbert does not consider the even branches of the conic and cubic as nested. Hilbert bases some of his investigations upon results previously obtained by A. Harnack,† and his method is entirely analogous to that of the latter.

Harnack had proved, in the article referred to, that a plane algebraic curve, without singularities, of order $n$ and of deficiency $p$, can not have more than $p+1$, that is, $\frac{1}{2}(n-1)(n-2)+1$ *real* branches; and, further, that, for every positive integral value of $n$, a non-singular curve with $\frac{1}{2}(n-1)(n-2)+1$ real branches actually exists. Setting out from this result of Harnack's, Hilbert shows first that a non-singular curve can have no more than $\frac{1}{2}(n-2)$ or $\frac{1}{2}(n-3)$ nested branches, according as $n$ is even or odd; for, if it had more, a right line could be drawn meeting the curve in more

* This definition is not scientific but it serves the present purpose. To make it rigorous Mr. Hilbert needs only to define accurately what is meant by *inside* and *outside* of a closed branch. Such a definition has virtually been given by von Staudt, *Geometrie der Lage*, § 1, 16.

† *Mathematische Annalen*, Bd. 10, *Ueber die Vieltheiligkeit der ebenen algebraischen Curven*