

turned the long controversy on the nature of force and energy between Descartes, Leibnitz, and their followers.\*

The closing section contains some interesting general remarks on the nature of the three laws and the ways of testing their truth.

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### WEIERSTRASS AND DEDEKIND ON GENERAL COMPLEX NUMBERS.

WEIERSTRASS †—*Zur Theorie der aus  $n$  Haupteinheiten gebildeten complexen Grössen. Göttingen Nachrichten, 1884.*

DEDEKIND—*Zur Theorie der aus  $n$  Haupteinheiten gebildeten complexen Grössen. Göttingen Nachrichten, 1885.*

DEDEKIND—*Erläuterungen zur Theorie der sogenannten allgemeinen complexen Grössen. Göttingen Nachrichten, 1887.*

IN closing his second memoir on biquadratic residues ‡ Gauss makes this remark: "Our general arithmetic, which goes so far beyond the limits of the geometry of the ancients, is entirely the creation of recent times. Starting with the notion of whole numbers its field has widened little by little. To whole numbers came fractions, to rational numbers the irrational ones; to the positive came the negative and to the real came the imaginary."

Once convinced that  $\sqrt{-1}$  was properly an algebraical quantity and that it had a meaning, mathematicians began to look for other quantities of a similar nature. "Why," they asked themselves, "should algebra yield an imaginary unit which makes it possible to represent two dimensions of space analytically; and fail to yield a second imaginary unit which can be used to represent the third dimension?" The thing needed only to be sought for apparently, and at first they looked amongst the functions of  $\sqrt{-1}$ . Unfortunately it turned out that even the most promisingly irrational of these could all be broken up into a real part and  $\sqrt{-1}$  times a second real quantity; algebra had done her best; if mathematicians wanted more imaginaries they must invent them. From the time of Gauss, then, until the present day the architects and the masterbuilders have turned occasionally

\* See for instance E. MACH, *Die Mechanik in ihrer Entwicklung*, Leipzig, Brockhaus, 1889, pp. 254-259.

† Extract from a letter to Schwarz.

‡ *Werke*, II., p. 175.