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# Parametric Feynman integrals and determinant hypersurfaces

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**Abstract**

The purpose of this paper is to show that, under certain combinatorial conditions on the graph, parametric Feynman integrals can be realized as periods on the complement of the determinant hypersurface $\hat{\mathcal{D}}_\ell$ in affine space $\mathbb{A}^{\ell^2}$, with $\ell$ the number of loops of the Feynman graph. The question of whether these are periods of mixed Tate motives can then be reformulated as a question on a relative cohomology of the pair $(\mathbb{A}^{\ell^2} \smallsetminus \hat{\mathcal{D}}_\ell, \hat{\Sigma}_{\ell,g} \smallsetminus (\hat{\Sigma}_{\ell,g} \cap \hat{\mathcal{D}}_\ell))$ being a realization of a mixed Tate motive, where $\hat{\Sigma}_{\ell,g}$ is a normal crossing divisor depending only on the number of loops and the genus of the graph. We show explicitly that the relative cohomology is a realization of a mixed Tate motive in the case of three loops and we give alternative formulations of the main question in the general case, by describing the locus $\hat{\Sigma}_{\ell,g} \smallsetminus (\hat{\Sigma}_\ell \cap \hat{\mathcal{D}}_\ell)$ in terms of intersections of unions of Schubert cells in flag varieties. We also discuss different methods of regularization aimed at removing the divergences of the Feynman integral.

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