Asian J. Math.
Vol. 15, No. 3, pp. 357–368, September 2011

002

IS THERE A NOTION OF WEAK MAXIMAL CONTACT IN CHARACTERISTIC $p > 0$?*

VINCENT COSSART†

Dedicated to Professor H. Hironaka

Key words. Desingularization, maximal contact, positive characteristic.

AMS subject classifications. 14E15.

Introduction. This article is a continuation of the conference made in the meeting "On the Resolution of Singularities" (December 2008) at the Research Institute for Mathematical Sciences (RIMS) of Kyoto and downloadable on their site.

The purpose of this conference is to give some hints about the proof of the theorem cited below and to show on two examples that the notion of maximal contact should be completely revised in the case of the positive characteristic.

Let us see the following theorem.

Theorem. (Cossart and Piltant) [CP1,CP2]. *Let k be a field of positive characteristic which is differentially finite over a perfect field k_0 and Z/k be a reduced quasiprojective scheme of dimension three with singular locus Σ. There exists a projective morphism $\pi : \tilde{Z} \to Z$, such that*
(i) $\tilde{Z}$ is regular.
(ii) π induces an isomorphim $\tilde{Z}\backslash\pi^{-1}(\Sigma) \simeq Z\backslash\Sigma$.
(iii) $\pi^{-1}(\Sigma) \subset \tilde{Z}$ is a divisor with strict normal crossings.
Such a $\tilde{Z}$ is called *a desingularization of Z*.

Two strategies. There are two main strategies to prove the existence of a desingularization.

The first one was initiated by O. Zariski in [Z]: he cutted the problem in two parts.
(1) Uniformization along a valuation,
(2) patching the uniformizations to get a desingularization.
This is the strategy used in [A1], [CP1, CP2], [Z].

The other strategy was initiated by H. Hironaka who introduced in [H] the key ideas and the fundamental techniques. This strategy is very fruitful in characteristic 0 and is followed by many others: Bierstone, Milman, Villamayor, Encinas, Hauser, Włodarczyk, Cutkosky, Temkin and all people I forget.

To simplify, we can say that the trick is to make a descending induction on the embedding dimension. At the very beginning, your singular variety Z is a closed subvariety of some regular variety W. Then, at every singular point $x \in Z$ there exists a closed *regular* subvariety $W_x \subset W$ which has maximal contact with Z at x. This notion is defined recursively (we are more precised in **III.2.1**):
(1) the Hilbert-Samuel stratum $HS-stratum(Z,x)$ of x is contained in W_x,

*Received June 16, 2010; accepted for publication May 20, 2011.
†Laboratoire de Mathématiques LMV UMR 8100, Université de Versailles, 45 avenue des États-Unis, 78035 Versailles cedex, France (cossart@math.uvsq.fr).