ASIAN J. MATH.
Vol. 15, No. 2, pp. 273–320, June 2011

005

FORMES DE WHITNEY ET PRIMITIVES RELATIVES DE FORMES DIFFÉRENTIELLES SOUS-ANALYTIQUES *

JEAN-PAUL BRASSELET † AND BERNARD TEISSIER ‡

Dédié à Heisuke Hironaka

Key words. Formes de Whitney, Whitney forms, sous-analytique, subanalytic, integration.

AMS subject classifications. 32B20, 32B25, 58C35.

0. English Abstract. The origin of this work is a question asked by François Treves to the second author in 1982 :

Given a real-analytic map $g\colon \mathbf{S}^n \to \mathbf{R}$, where $\mathbf{S}^n$ is the n-dimensional sphere, and a differential r-form ω of class $\mathcal{C}^\infty$ on $\mathbf{S}^n$ whose restriction to every non-singular fiber is exact, does there exist a hölderian differential $(r-1)$-form Ω on $\mathbf{S}^n$ such that

$$dg \wedge (\omega - d\Omega) = 0,$$

where the differential $d\Omega$ is taken in the sense of distributions ?

We prove an analogue of this statement for a continuous subanalytic form ω, the existence of a subanalytic continuous relative primitive Ω satisfying $dg \wedge (\omega - d\Omega) = 0$ under the assumption that the restriction of ω to each non singular fiber is the differential in the sense of distributions of a subanalytic form, in the more general framework of triangulable subanalytic maps $g\colon X \to \mathbf{R}^n$ between non singular spaces. (see Corollaire 6.3 below).

Recall that Masahiro Shiota has proved that every subanalytic morphism from a compact space to $\mathbf{R}$ is triangulable (see [Sh1], [Sh2], Chap. II, §3) and that the second author has proved (see [Te1]) that every proper subanalytic map becomes triangulable locally on the base after base changes that are finite compositions of local blowing-ups. S. Chanillo and F. Treves proved in 1997 (see [C-T], Lemma 2.2) a result analogous to the statement above : under the hypothesis of vanishing of ω to infinite order along the singular fibers of g, they obtain a relative primitive of class $\mathcal{C}^\infty$ satisfying the same vanishing condition.

Recall also that a function h on an analytic manifold U is said to be hölderian if each point of U has a neighborhood V such that there exist positive constants α and C such that for $x, x' \in V$ one has $|h(x) - h(x')| \leq C|x - x'|^\alpha$. A differential form on U is said to be hölderian if its coefficients are hölderian functions on U and its differential in the sense of distributions can be represented by a form with hölderian coefficients. Recall finally that for a function, to be subanalytic and continuous implies the Hölder property thanks to the Łojasiewicz inequalities extended to the subanalytic case by Hironaka (*cf* [Hi], [Ha] and §1).

The main idea is to transform this problem of analysis into a problem of geometry by representing differential forms by means of Whitney forms. Recall that the differential form which Whitney associates (*cf* [Whi, Chap. IV, §27]) to a simplex of a

*Received November 15, 2010 ; accepted for publication May 20, 2011.

†Institut de Mathématiques de Luminy, UMR 6206 du CNRS, Campus de Luminy, Case 907– 13288 Marseille Cedex 9, France (jpb@iml.univ-mrs.fr).

‡Equipe Géométrie et Dynamique, Institut Mathématique de Jussieu, UMR 7586 du CNRS, 175 rue du Chevaleret, 75013 Paris, France (teissier@math.jussieu.fr).