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COEFFICIENT AND ELIMINATION ALGEBRAS IN RESOLUTION OF SINGULARITIES*

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Dedicated to H. Hironaka on his 80th birthday

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Introduction. Given a variety X over a field k one wants to find a desingularization, which is a proper and birational morphism $X' \to X$, where X' is a regular variety and the morphism is an isomorphism over the regular points of X.

If X is embedded in a regular variety W, there is a notion of embedded desingularization and related to this is the notion of log-resolution of ideals in $\mathcal{O}_W$.

When the field k has characteristic zero it is well known that the problem of resolution is solved. The first proof of the existence of resolution of singularities is due to H. Hironaka in his monumental work [Hir64] (see also [Hir77]).

If characteristic of k is positive the problem of resolution in arbitrary dimension is still open. See [Hau10] for recent advances and obstructions (see also [Hau03]).

The proof by Hironaka is existential. There are constructive proofs, always in characteristic zero case, see for instance [VU89], [VU92], [BM97], we refer to [Hau03] for a complete list of references. Those constructive proofs give rise to algorithmic resolution of singularities, that allows to perform implementation at the computer [BS00], [FKP04].

Recently some techniques have appeared in order to try to prove the problem of resolution of singularities in the positive characteristic case. Rees algebras seem to be a useful tool in this context. Hironaka in [Hir03] and [Hir05] propose to use Rees algebras for proving log-resolution of ideals. The advantage of Rees algebras is that the algebra encodes in one object many ideals which are "equivalent" for the problem of log-resolution. Also Rees algebras have a good behavior with respect to integral closure, see for instance [VU08] and [VU07]. On the other hand Kawanoue and Matsuki [Kaw07], [KM06] use a different object, called idealistic filtration, which is similar to Rees algebras but with a grading over the real numbers.

In this paper we compare those structures and construct $\mathbb{Q}$-Rees algebras (1.7), which are algebras with grading over the rational numbers. We will see that Rees algebras, idealistic filtrations and $\mathbb{Q}$-Rees algebras encode (up to integral closure) the same information (1.15). One motivation to extend Rees algebras to a $\mathbb{Q}$-grading comes from the scaling operation (3.15), which is needed in the process of resolution of singularities. Since we restrict to rational numbers all properties related to integral closure and finiteness come easily, see 1.6.

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