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ESTIMATES FOR THE HEAT KERNEL ON DIFFERENTIAL FORMS ON RIEMANNIAN SYMMETRIC SPACES AND APPLICATIONS *

N. LOHOUÉ† AND S. MEHDI‡

Abstract. We prove upper bounds estimates for the large time behavior of the heat kernel and for the resolvent of the form Laplacian on Riemannian symmetric spaces, and we obtain $L^{2+\epsilon}$-estimates for its resolvent on locally symmetric spaces. We deduce lower bounds for the bottom of the spectrum of the form Laplacian and some results on the vanishing of the L^2-cohomology of locally symmetric spaces.

Key words. Heat kernel, Form Laplacian, Riemannian symmetric space, locally symmetric space, Plancherel formula, Paley-Wiener theorem, L^2- cohomology, principal series representation, isotropy representation.

AMS subject classifications. Primary 22E46, 43A85, 53C35, 58J35, 58J50; Secondary 22E40, 57T15

1. Introduction. In the last decades the heat kernel has become a fundamental and powerful tool, subject of a rich and vast literature, reflecting its universality and formidable efficiency: Atiyah-Singer index theory, K-theory, spectral geometry, zeta and theta functions, L^2-invariants, anomalies, quantum gravity, ... (see e.g. [4], [7], [28], [37]). Despite these tremendous advances, explicit estimates for the asymptotics of the heat kernel and computation of related L^2-invariants are not available in general. However, for a large class of Riemannian manifolds, representation theoretic techniques may be used to obtain estimates for the asymptotics, compute L^2-invariants and derive some results on the L^2-cohomology.

More precisely, let G be a non compact connected semisimple Lie group with finite center and K a maximal compact subgroup of G. The homogeneous space G/K is naturally equipped with a structure of a non compact Riemannian symmetric manifold, the metric being induced by the Killing form of G. A finite dimensional representation (τ, E) of K induces a homogeneous vector bundle $\mathcal{E}$ over G/K. The group G acts by left translations on the Hilbert space $L^2(G/K,\mathcal{E})$ of square integrable sections of $\mathcal{E}$. Let

$$D : L^2(G/K,\mathcal{E}) \to L^2(G/K,\mathcal{E})$$

be a G-invariant selfadjoint positive elliptic operator, i.e D commutes with the action of G on $L^2(G/K,\mathcal{E})$. Denote by $P_t = e^{-tD}$ the fundamental solution of the heat equation

$$\begin{cases} DP_t = -\frac{\partial}{\partial t}P_t, \quad t > 0 \\ P_0 = \delta \end{cases}$$

where δ is the Dirac function. In particular, for each ψ in $L^2(G/K,\mathcal{E})$, the convolution

*Received May 30, 2009; accepted for publication October 4, 2010.
†UMR 8628–CNRS and Département de mathématiques, Université Paris Sud–Orsay, France (Noel.Lohoue@math.upsud.fr).
‡UMR 7122–CNRS and Département de mathématiques, Université Paul Verlaine–Metz, France (mehdi@univ-metz.fr).