

EMBEDDED CONSTANT MEAN CURVATURE HYPERSURFACES ON SPHERES*

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Abstract. Let $m \geq 2$ and $n \geq 2$ be any pair of integers. In this paper we prove that if H lies between $\cot(\frac{\pi}{m})$ and $b_{m,n} = \frac{(m^2-2)\sqrt{n-1}}{n\sqrt{m^2-1}}$, there exists a non isoparametric, compact embedded hypersurface in S^{n+1} with constant mean curvature H that admits $O(n) \times Z_m$ in its group of isometries. These hypersurfaces therefore have exactly 2 principal curvatures. When $m = 2$ and H is close to the boundary value $0 = \cot(\frac{\pi}{2})$, such a hypersurface looks like two very close n -dimensional spheres with two catenoid necks attached, similar to constructions made by Kapouleas. When $m > 2$ and H is close to $\cot(\frac{\pi}{m})$, it looks like a necklace made out of m spheres with $m + 1$ catenoid necks attached, similar to constructions made by Butscher and Pacard. In general, when H is close to $b_{m,n}$ the hypersurface is close to an isoparametric hypersurface with the same mean curvature. For hyperbolic spaces we prove that every $H \geq 0$ can be realized as the mean curvature of an embedded CMC hypersurface in H^{n+1} . Moreover we prove that when $H > 1$ this hypersurface admits $O(n) \times Z$ in its group of isometries. As a corollary of the properties we prove for these hypersurfaces, we construct, for any $n \geq 6$, non-isoparametric compact minimal hypersurfaces in S^{n+1} whose cones in \mathbf{R}^{n+2} are stable. Also, we prove that the stability index of every non-isoparametric minimal hypersurface with two principal curvatures in S^{n+1} exceeds $n + 3$.

Key words. Constant mean curvature, embedded, principal curvatures.

AMS subject classifications. 53C42, 53A10

1. Introduction. Minimal hypersurfaces of spheres that have exactly two principal curvatures at each point were initially studied by Otsuki in [14]. He reduced the problem of classifying them, to that of solving an ODE, and the problem of deciding about their compactness, to the problem of studying an integral that relates periods of two functions involved in the immersions that he found. For surfaces in \mathbf{R}^3 , Delaunay in 1841 [6] showed that if one rolls a conic section on a line in a plane and then rotates about that line the trace of a focus, one obtains a CMC surface of revolution. CMC stands for constant mean curvature. This rolling construction was generalized for the case of CMC hypersurfaces in \mathbf{R}^{n+1} by Hsiang and Yu in the early eighties [10], [11] and for CMC hypersurfaces in the hyperbolic space and the sphere by Sterling in 1987 [21].

After Otsuki's paper in 1970, several properties for a CMC hypersurface $M \subset S^{n+1}$ with exactly two principal curvatures were proved in [7], [4], [17], [22], [9], [1], [2], [23], [24], [25], and [13] among others.

For the case $n = 2$, we give explicit trigonometric formulas for immersions of CMC hypersurfaces in S^3 . A gallery of pictures of the stereographic projection of some of these surfaces, made by Schmitt, can be found in the GANG (Geometry Analysis Numerics Graphics, University of Massachusetts) web page. These surfaces are called unduloidal tori in S^3 with m -lobes because all of them have Z_m , for some m , in their group of symmetries. In this paper we will prove that this symmetry property holds in every dimension and we will also prove that for every positive integer m , if H lies between

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