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001

ON LEVEL-RAISING CONGRUENCES*

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Abstract. In this paper we rewrite a work of Sorensen to include nontrivial types at the infinite places. This work extends results of K. Ribet and R. Taylor on level-raising for algebraic modular forms on $D^\times$, where D is a definite quaternion algebra over a totally real field F. We do this for any automorphic representations π of an arbitrary reductive group G over F which is compact at infinity. We do not assume π_∞ is trivial. If λ is a finite place of $\bar{\mathbb{Q}}$, and w is a place where π_w is unramified and $\pi_w \equiv \mathbf{1} \pmod{\lambda}$, then under some mild additional assumptions (we relax requirements on the relation between w and ℓ which appear in previous works) we prove the existence of a $\tilde{\pi} \equiv \pi \pmod{\lambda}$ such that $\tilde{\pi}_w$ has more parahoric fixed vectors than π_w. In the case where G_w has semisimple rank one, we sharpen results of Clozel, Bellaiche and Graftieaux according to which $\tilde{\pi}_w$ is Steinberg. To provide applications of the main theorem we consider two examples over F of rank greater than one. In the first example we take G to be a unitary group in three variables and a split place w. In the second we take G to be an inner form of GSp(2). In both cases, we obtain precise satisfiable conditions on a split prime w guaranteeing the existence of a $\tilde{\pi} \equiv \pi \pmod{\lambda}$ such that the component $\tilde{\pi}_w$ is generic and Iwahori spherical. For symplectic G, to conclude that $\tilde{\pi}_w$ is generic, we use computations of R. Schmidt. In particular, if π is of Saito-Kurokawa type, it is congruent to a $\tilde{\pi}$ which is not of Saito-Kurokawa type.

Key words. Level-Raising, Congruences, Algebraic Modular Forms.

AMS subject classifications. 11F33, 22E55, 11F70, 11F85, 11F46, 20G25, 22E35

Introduction. This paper stems from the following result of Ribet [R]:

THEOREM 0.1. *Let $f \in S_2(\Gamma_0(N))$ be an eigenform. Let $\lambda|\ell$ be a finite place of $\bar{\mathbb{Q}}$ with $\ell \geq 5$ and f not congruent to an Eisenstein series modulo λ. Let q be a prime number with $(q, N\ell) = 1$ and $a_q(f)^2 \equiv (1+q)^2 \pmod{\lambda}$. Then there exists a q-new eigenform $\tilde{f} \in S_2(\Gamma_0(Nq))$ congruent to f mod λ.*

Two eigenforms f and $\tilde{f}$ are said to be *congruent modulo* λ if their Hecke eigenvalues are algebraic integers congruent for almost all primes, that is, if $a_p(f) \equiv a_p(\tilde{f}) \pmod{\lambda}$ for almost all p. The proof of this theorem can be reduced, via the correspondence from an inner form to GL(2) (see [F] for a simple proof), to the corresponding statement for $D^\times$ where D is a definite quaternion algebra over $\mathbb{Q}$.

A goal of this paper is to prove that an automorphic form of Saito-Kurokawa type is congruent to an automorphic form which is *not* of Saito-Kurokawa type. By functoriality ([F1]) the statement can be reduced to that for an inner form G of $\mathrm{PGSp}(2)/F$ such that $G(\mathbb{R})$ is compact. Indeed, the set of packets of automorphic representations of $G(\mathbb{A})$ can be identified with a subset of the set of such objects on $\mathrm{PGSp}(2,\mathbb{A})$, where almost all local components are the same. By a form on $G \simeq \mathrm{SO}(5)$ of Saito-Kurokawa type we mean the lift of $\mathbf{1} \times \rho$ from the endoscopic group $\mathrm{PGL}(2,\mathbb{A}) \times \mathrm{PGL}(2,\mathbb{A})$ to $\mathrm{PGSp}(2,\mathbb{A})$, where ρ is cuspidal and $\mathbf{1}$ is trivial on $\mathrm{PGL}(2,\mathbb{A})$. It is nontempered at almost all places. We achieve this goal in Theorem 0.6, proven in Section 12.

We apply ideas and methods of R. Taylor [T1] and [T2]. The level-raising part of Taylor's proof carries over to the following more general setup. Let F denote a totally real number field with ring $\mathbb{A} = F_\infty \times \mathbb{A}^\infty$ of adèles. We denote the set of real places

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