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LOCAL HARNACK ESTIMATE FOR MEAN CURVATURE FLOW IN EUCLIDEAN SPACE*

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Abstract. We obtain the local Harnack estimate of mean curvature flow in Euclidean space $\mathbb{R}^{n+1}$, under the condition $-m(t)g_{ab} \leq h_{ab} \leq Mg_{ab}$, s.t. $0 \leq m(t) \leq M$, and $D_t m(t) \geq (n+3)mM^2$, on $t \in [0, \frac{\pi^2}{4(n+1)M^2}]$. As a corollary, we get a sharp gradient estimate of mean curvature in some directions.

Key words. Mean curvature flow, Local Harnack estimate.

AMS subject classifications. 53C21, 53C44

1. Introduction. The differential Harnack estimate of mean curvature flow was done by R.Hamilton in [1]. Recently, Hamilton find a new method to get local Harnack inequality for Ricci flow in [2].

THEOREM 1. *(Hamilton's Local Harnack estimate for Ricci-flow) Let M^n is a Riemannian manifold. $(M, g(t))$ is the solution of the Ricci-flow equation*

$$\frac{\partial}{\partial t} g_{ij} = -2R_{ij}, \qquad t \in [0, T).$$

$U \subset M^n$ is an open set. On $U \times [0, t_0]$, $t_0 < T$ it satisfies the following curvature condition: $\exists C_0$, $\forall M > 0$, where M is a positive constant

$$\begin{cases} -m(t)(g_{ac}g_{bd} - g_{ad}g_{bc}) \leq R_{abcd} \leq M(g_{ac}g_{bd} - g_{ad}g_{bc}), & \\ 0 \leq m(t) \leq M & t \in [0, t_0] \\ m'(t) \geq C_0 mM & t \in [0, t_0] \end{cases}$$

$O \in U$, set $C_1 = Mr^2$, s.t.$B_r(O, t_0) \subset\subset U$, then for $\forall (p, t) \in B_{\frac{r}{2}}(O, t_0) \times [t_0 - \frac{r^2}{4}, t_0]$ and $\forall V \in T_pM^n$, we have

$$DR(V)^2 \leq CM^2(Rc(V, V) + Cm|V|^2).$$

Where C only depends on n, C_1 is a positive constant.

Then by using the inequality, Hamilton get a theorem of curvature bound at finite distance for Ricci-flow in [3].

Motivated by his work, the author do a similar work in mean curvature flow. Maybe the work will be used in mean cuvature flow as the same way.

Let M^n be a smooth manifold without boundary, and let F_0: $M^n \to \mathbb{R}^{n+1}$ a smooth immersion. Let

$$F(\cdot, t) : M^n \times [0, T) \to \mathbb{R}^{n+1},$$

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